LAN-ALOG

ANALOGUE COMPUTER HANDBOOK

by

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L A N - A L O G   C O M P U T E R   H A N D B O O K

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N O T E !!

The amplifiers used in LAN-ALOG are high gain types and if the machine is switched on with no feedback components patched in, the overload lamps will light.

This does not indicate a fault in LAN-ALOG and will not damage the machine.

When using the machine always ensure either the resistive or capacitive feedback component is patched in on all amplifiers. Refer to the LAN-ALOG handbook (page 1) for full details.

LAN-ALOG TYPE LA-4R

POT SET OPERATING INSTRUCTIONS.

The LA-4R is a modified version of the LA-4, having a ten turn helicalical reference potentiometer of 0.25% linearity. Additional colour coded sockets are situated on the patch panel adjacent to the reference pot, and connections are as follows:

NORMAL OPERATION - VOLTMETER.

When the VOLTMETER is used for monitoring the output from the operational amplifiers, or, to set the POWER UNIT ± 10 and ± 15 outputs, patching is as follows:

a) Y blue meter socket is the input to the meter. A green patch cord should be inserted to enable any amplifier or power unit output socket to be connected to the meter.

b) Connect Blue socket Bt to Green socket Bv, thus earthing one side of the meter.

POT SET USING THE REFERENCE POT.

For POT SET the VOLTMETER is used as a nulling indicator in conjunction with the REFERENCE POT. Patching instructions as follows:

a) Remove patch cords from Blue socket Bt and Green socket Bv if the meter has been used for voltage measurement.

b) Connect Blue socket Bt to Yellow socket Bs (reference pot slider). Connect Yellow socket Ay (top end of reference pot) to either the + 10V or minus 10V reference voltage sockets Aw or Ax.

c) Connect a Green patch cord to meter socket Y Blue (situated immediately above the meter) the free end of this cord may now be connected to the slider of any co-efficient pot. The co-efficient pot must be connected from C socket to the desired reference voltage polarity (a or b sockets in columns A.B.C.D. or W or X sockets below pot 5 and 6). The green patch cord being connected to the co-efficient pot slider.

d) Adjust the REFERENCE POT to the desired setting and rotate the co-efficient pot until the VOLTMETER reads centre zero. Press the 1.5V meter range button to increase sensitivity in order to achieve accurate null balance.

The co-efficient pot is now set to the REFERENCE POT.

PREFACE

The Lan-Alog unit has been designed to teach analogue computer techniques to a wide range of students in mathematics, engineering and applied science. The computer contains four encapsulated operational amplifiers, employing silicon epitaxial transistors and a good quality voltmeter. There are very flexible arrangements for inter-connecting the amplifiers, to handle a variety of problems and demonstrations, particularly those involving first and second order differential equations.

The amplifier fitted in the standard version of Lan-Alog has an open-loop gain of 50,000, so ensuring a basic computing accuracy of better then 1% per amplifier. The overall accuracy of computation depends also on the quality of the resistors and capacitors employed, but should be better than $\pm$ 3% in all conditions, for one Lan-Alog. Amplifiers having a gain of 5×10^5 and higher quality components are available at extra cost.

It is intended that the d.c. supply voltages be derived from a mains operated stabilized power unit and a compact power unit capable of supplying two Lan-Alog computers is available.

Several members of the staff at Enfield College and Lan-Electronics have contributed valuable ideas during the development of Lan-Alog.

We wish to acknowledge the permission of the Governors of Enfield College (through the Head of the Electrical Engineering Department) for this handbook to be published.

Enfield.

J. R. A.
J. R. L. R.

CHAPTER 1.

INTRODUCTION TO LAN-ALOG.

1.1. Layout of the Computer.

All the components of the Lan-Alog unit are mounted on a steel panel, measuring 19 x $10\frac{1}{2}$ inches and the panel is drilled such that it may be fixed vertically on a standard 19-inch rack.

The front of the computer panel is divided into four "columns", labelled A, B, C, and D. Each column includes one operational amplifier, indicated by a symbol similar to that of Figure 1.1., where the column letter is in the amplifier symbol.

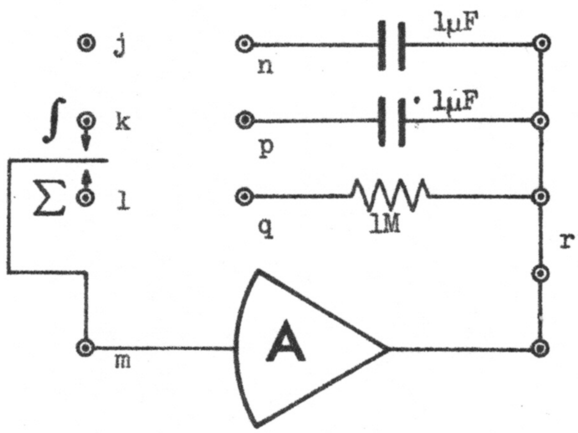


FIG.1.1. Dual-purpose Operational Amplifier.

The output of each amplifier is brought to five sockets, on the right, which are joined in common and labelled 'r' on the panel. Any of the four amplifiers may be used with either capacitive or resistive feedback, to give either integration or summing. This is selected by the toggle switch, marked with the symbols $\int$ (for Integrate) and Σ (for Sum) above each amplifier symbol.

Three separate input resistors, two of 1MΩ and one of 100kΩ are to the left of each amplifier. These resistors are connected to a "summing junction" consisting of three sockets labelled 'h'. In most cases a summing junction will be joined directly to the input of the associated amplifier, at point 'm'. However, if need be, two summing junctions may be taken to the input of one amplifier. Socket 'j' is provided as the input point for an initial condition voltage, if this is required at the feedback capacitor.

One potentiometer is provided for each amplifier to set the output to zero, when the computer control is in the BALANCE position. These potentiometers, marked A,B,C and D, are on either side of the voltmeter. Six coefficient potentiometers are in columns A and B. Four potentiometers have one terminal permanently connected to the earth (OV) line, with two sockets 'c' for the input and the output, or "Slider", brought to a pair of sockets 'd'. Potentiometers 5 and 6 are provided with a socket 's' and 'u' at either end of the resistance and a pair of sockets 't' joined to the slider. This gives considerable flexibility in use, as the two double-ended potentiometers may be used in either series or shunt.

The power unit includes two Reference Voltage potentiometers, which are intended to adjust the positive and negative supplies to exactly 10V. These stabilized Reference Voltages are available at the sockets 'a' (+ 10) and 'b' (- 10) in each column and at 'w' (+ 10) and 'x' (- 10) in columns A and D.

A centre-zero voltmeter, having a resistance of 10,000 Ω/V is mounted in column B and C. The input to this meter is through the socket 'y', above the meter scale. Only a single input is provided to the meter, as one terminal is joined permanently to the earth line within the computer. The voltemter is usually set to read ± 15V and may be switched to either the ± 1.5V or ± 0.15V range by operating the appropriate push-button, below the meter scale.

The common operating controls are in columns C and D. The power switch and associated red indicator lamp are at the lower right hand side, together with the five-way plug for the power unit connector. There are two 3-position switches to select the operating mode. One is marked Master - Balance - Slave and the other Reset - Compute - Hold. The Auto-Manual toggle switch gives the choice of repetitive or "single-shot" operation. In the Auto mode the computer may be operated at speeds from one computation per ten seconds to about 25 operations per second. This frequency is controlled by a transistor multivibrator having two speed ranges (selected by the Slow - Fast toggle switch) and a speed control potentiometer, which has 10 nominal scale markings. The Automatic Reset Time is 1 second in the Slow range and equal to the Compute Time on the Fast range. A seven-way socket is provided for the synchronizing connector to join two Lan-Alog units, if more than four amplifiers are required for one computation.

Two overload indicator lamps are provided in the centre of the computer panel. These operate when the voltage in any part of the computing circuit exceeds ± 10V. A single change-over contact, on the multivibrator operated relay, is brought to sockets 's', 't', 'u' in each of columns B and C. These two contacts may be useful in certain operations.

1.2. Inputs and Outputs

This analogue computer requires stable d.c. supplies of ± 15V and ± 10 V. with a common earth line at OV. The current drain is about $\frac{1}{4}$ A at each of these two voltages. Lan-Electronics manufacture a compact mains-operated power unit which is designed to supply one or two Lan-Alog computers, which must be used for this computer.

For certain problems it is necessary to introduce a forcing function, which is in the form of a voltage wave from an external function generator. This waveform may be fed into the input of any of the four operational amplifiers as required by the problem. Under no circumstances should any peak voltage of greater than 10V (this means 7V r.m.s. with alternating current) be applied at any point in the amplifier circuits. The spare relay contacts (mentioned above) may be used to switch this forcing function in synchronism with the Compute and Reset operations.

The output voltage from any amplifier in the computer may be observed by one of three possible methods. For steady state problems, and for an approximate check on dynamic problems, the output voltage may be read off the Lan-Alog voltmeter. For demonstrating the results of dynamic problems an oscilloscope is the more suitable form of output indicator. A cathode ray oscilloscope with a good d.c. response, a very low frequency timebase and tube with long persistence is desirable. Good results will be obtained with the four-channel display oscilloscope, having 19-inch screen, available from Lan-Electronics. If the computer is operated in the Auto mode at the maximum speed (about 25 c/s) then a simple oscilloscope with a standard time-base may be satisfactory. When a permanent record of the computer output is required, then some form of continuous paper recorder may be employed as an alternative to using a camera with an oscilloscope.

In most cases the user of Lan-Alog will find it convenient to operate the computer in the Auto, or repetitive, mode. Then it is desirable to synchronize the horizontal sweep of the oscilloscope with the repetition rate of the computer. This may be done by connecting the trigger sockets (in column A) to the Trigger, or Sync. input of the oscilloscope. The train of trigger pulses is derived directly from the controlling relay.

For some more complex problems it is necessary to operate two Lan-Alog units as one computer. This may be done by linking the two units with the special synchronizing cable, which is plugged into the 7-way socket in column D, and by switching one unit to Slave and the other to Master.

1.3. Operating the Controls.

After the power supply lead has been connected, the Power Switch, at the extreme right-hand of the unit, should be operated, to bring the red indicator lamp on. At the start of each period of use each amplifier in the Lan-Alog should be checked for zero balance. For this purpose the 3-position switch in column D must be put to Balance. The output point of each amplifier 'r' should be connected, in turn, to the Meter input 'y' and the appropriate Balance Potentiometer adjusted to give a voltmeter reading which is exactly zero. For this purpose Balance must be completed on the 0.15V range.

When one Lan-Alog unit is used on its own, or is the controlling unit of a pair, then the column D switch must be put to Master. In the event that the unit is the dependent one of a pair, then this switch will be put to Slave. It should be noted that two Lan-Alog computers may be operated as independent units from a common power supply, if both are switched to Master.

If the Auto/Manual toggle switch in column D is in the Manual position then the computer is under the control of the 3-position switch, which must be moved from Compute to Reset and back for each complete cycle of operation. The computation may be stopped at any instant when under Manual control, by putting this 3-position switch to Hold. When the toggle switch is put to Auto, and the 3-position control to Compute, then the compute-reset cycle proceeds automatically, at a rate determined by the position of the Slow/Fast switch and the Speed Control potentiometer.

The approximate setting of each of the six linear coefficient potentiometers is indicated by the ten scale marks around the potentiometer knob. These potentiometers are usually employed as a voltage step-down control at the input of the associated amplifier. A potentiometer may be supplied either from ±10V supply or from the output of an amplifier. If the input is from the ±10V Reference supply then the potentiometer slider (points'd'or't') is joined to the Meter input and the required coefficient is set. For example, if the desired coefficient is 0.58, then the meter reading should be 0.58 x 10V, that is 5.8V. If a potentiometer is to be supplied from an amplifier output then to set the coefficient the potentiometer input must be temporarily disconnected from the amplifier and connected instead to the 10V. The setting is made as above and the original patching from the amplifier output then replaced. It is good proctice to set potentiometers with their loads connected.

It will be noticed that both ends of potentiometers 5 and 6 are brought out to sockets 's'and'u'. There is an occasional requirement for a potentiometer to be used "floating" (i.e. with neither end earthend).

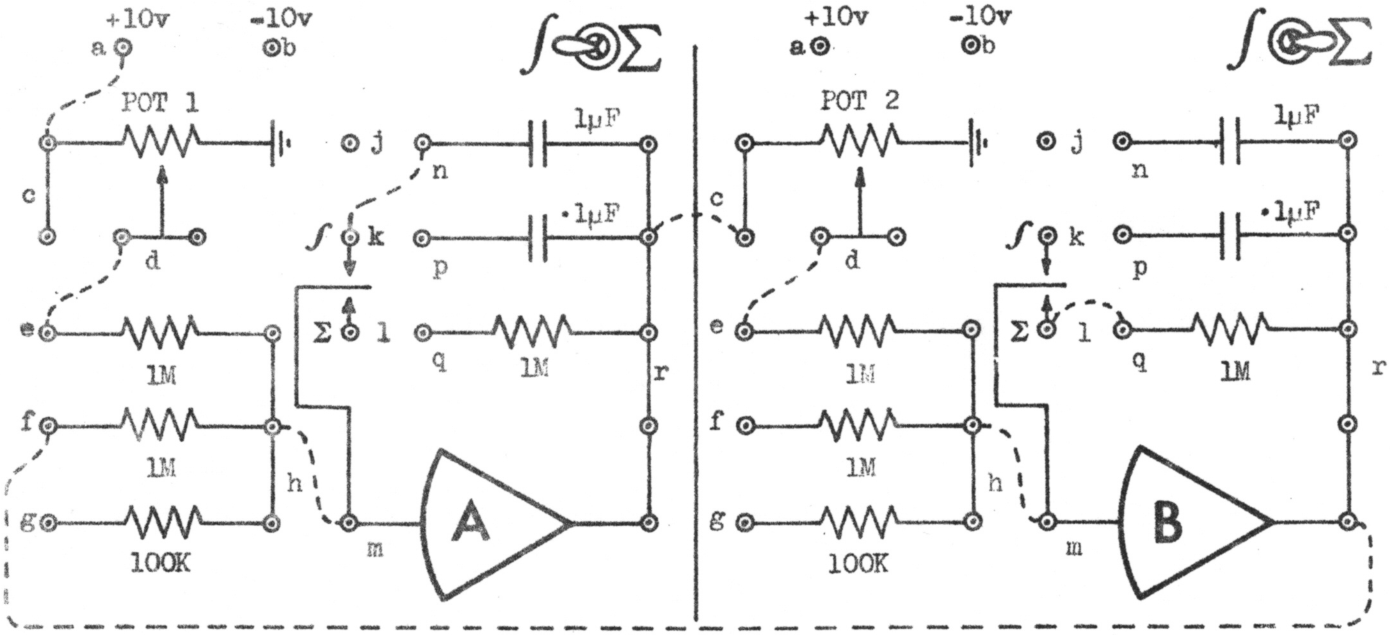
To set the coefficient in this case one end must be patched temporarily to earth 'v' and the other end of the potentiometer to a 10V supply. These temporary patching cords must be replaced when the coefficent has been adjusted with the use of the meter.

1.4. Patching the Computer.

Each amplifier in the Lan-Alog unit may be used, without an input resistor or any feedback, simply as a high-gain d.c. amplifier. A good example of this is in a four-channel temperature recording system. For all the demonstrations and problems given in this handbook some circuit interconnections must be added on the front of the Lan-Alog panel. This is known as patching, (or the computer programme). To simplify the patching procedure each circuit access point on the Lan-Alog is identified by a small (lower case) letter. Any point can thus be specified by a two letter code, such as Aa, incorporating the large letter (within the amplifier symbol) of the column and the **small** letter for the socket position. The wiring programme, as an instruction to inter-connect two points can be written in the form As - Ac, for a direct inter connection. Occasionally it may be necessary to employ a patching cord containing a component , such as a resistor, in which case the inter-connection would be written as Ar -(200 kΩ resistor)- Bh, with the resistor value indicated.

The use of this wiring code considerably simplifies the instructions for any demonstration or experiment on the Lan-Alog. The programme may be written down from a simply block diagram of the circuit and also the wiring on the Lan-Alog is easily checked from the programme.

Figure 1.2. shows part of the Lan-Alog front panel with some wiring in position and the associated patching code.



Programme:	Aa - Ac,	An - Ak,	Bh - Bm,
	Ad - Ae,	Ar - Bc,	Bl - Bq,
	Ah - Am,	Bd - Be,	Br - Af.

FIG. 1.2. Wiring programme for a small analogue circuit.

For the specific programme of Figure 1.2. amplifier A must be switched to ∫ and amplifier B to Σ . Points'n'or'p'must always join to'k'(for Integration) and point'q'only to'l'(for Summing and sign changing). It should also be noted that some points in the computer appear at several sockets which are joined permanently together. These points may, therefore, appear the corresponding number of times in the wiring programme, for any one computer circuit. Any amplifiers not used in the computation must be switched to Σ . and patching cords plugged between sockets'h'and'm'and'l'and'q'for that amplifier.

Two sets of Tie sockets are provided, in case the number of sockets available at any stage of a programme is insufficient.

Sufficient patching cords are supplied with each Lan-Alog computer for any programme which may be required to solve a linear problem. If non-standard capacitance or resistance values are required, then external boxes may be used, together with the patching cords having a plug at one end only. A non-standard capacitance must be joined between points 'k' and 'r', of the relevent amplifier, and a resistance between 'l' and 'r'.

Patching cords incorporating semiconductor diodes are also available.

CHAPTER 2.

BASIC ANALOGUE COMPUTING.

2.1. The Operational Amplifier: Addition and Subtraction

The heart of any Electronic Analogue Computer is the High Gain Operational Amplifier. This may be regarded as a device which:-

(i) amplifies its input signal by a large factor (50,000 in the case of the Standard Lan-Alog amplifiers);

(ii) produces an output signal of opposite polarity to the input;

(iii) takes negligible input current.

Such an amplifier might be shown diagrammatically as in Fig. 2.1.(a) but since all signals in Analogue Computing are measured with reference to earth potential, the earthed lines are not necessary and are usually omitted as in Fig. 2.1.(b).

(a) Fig. 2.1. (b)

If the maximum output voltage is 10V and the gain is 50,000 as is the case in the Lan-Alog, then clearly the input point cannot differ from earth potential by more than 0.2mV (0.0002V) in normal working. To the accuracy of operation of the computer, this small voltage can be desregarded: the input point is considered to be at zero potential and it is described as a "virtual earth". It is the base terminal of the first transistor of the Lan-Alog transistor amplifiers and would be the grid terminal of the first valve in a thermionic amplifier: it is usually known by these names in order to distinguish it from the "input" of Fig. 2.2.

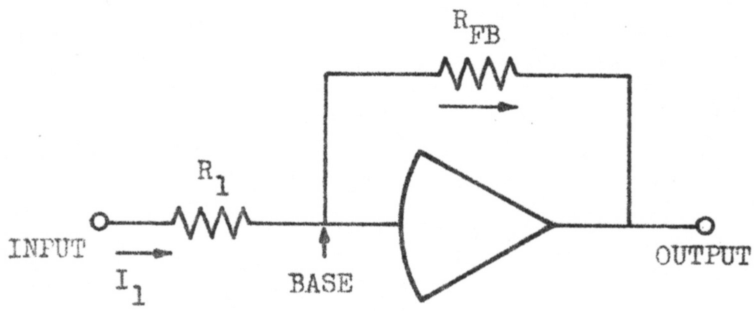


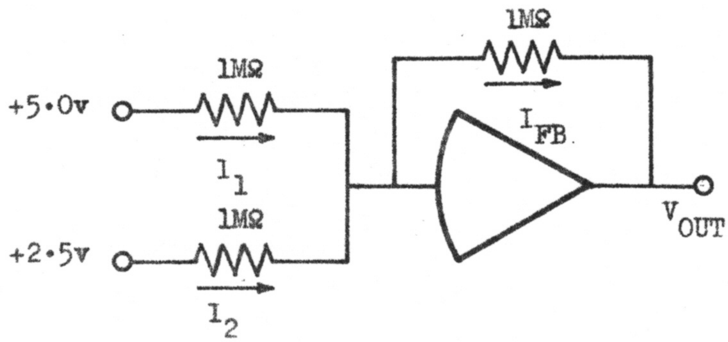
Fig 2.2.

Suppose two resistors R_1 and R_{FB} (F.B. standing for feedback) are connected as shown in Fig. 2.2., the value of each resistor being 1MΩ (1,000,000 ohms). Further suppose that a signal of + 5V is applied to the terminal "Input". Then, since the base potential is zero effectively, a current I_1 of $5/10^6$ A or 5μA will flow from left to right in R_1. Evidently 5μA must also flow in R_{FB} as shown by the arrow, since no current can enter the amplifier itself. Since R_{FB} is also 1MΩ the potential of the output terminal must be 5V negative with respect to the base, which means 5V negative with respect to earth also. Were this not so, the base would change slightly in potential and the amplifier would amplify until the condition <u>is</u> fulfilled.

All that has been achieved is a sign reversal, but by a simple extension, to the case of having more than one input, it will be seen that addition and subtraction of voltages is also possible.

Thus in Fig. 2.3.

$I_1 = 5.0\mu A$
$I_2 = 2.5\mu A$
$I_{FB} = 7.5\mu A$
and $V_{OUT} = -7.5V$



If the 2.5V input had been negative, I_2 would have been reversed, I_{FB} would have been only 2.5μA and the output voltage only -2.5V.

Mathematically the computation which has been achieved is that

$$V_{OUT} = -(V_1 + V_2 + \ldots\ldots.)$$

The signals can be added in varying proportions by altering the values of the input resistors. Thus in Fig. 2.4. $V_{OUT}=(V_1+10V_2)$

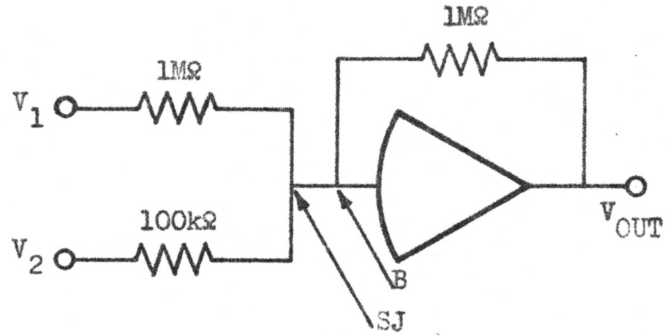


Fig. 2.4.

In this figure the point S J, where the input currents meet, is known as the "Summing Junction".

The summing junction and the base, B, are coded 'h' and 'm' respectively on the Lan-Alog panel. They are linked together in normal operation and form the virtual earth referred to above.

In drawing Computer Flow Diagrams, the circuits of Figs.2.3 and 2.4 are simplified as in Fig. 2.5 where the triangles indicate all the circuitry of a summing amplifier and the small figures indicate the proportions in which the input signals are added.

Fig. 2.5.

2.2. Multiplication by a Constant.

The circuit of Fig. 2.6 is used to produce the quantity kx voltage V_{IN}, which is proportional to 'x', is applied to the top end (as it is called) of a potentiometer, the other end of which is earthed. The slider is set for 'k' (where $0 < k < 1$).

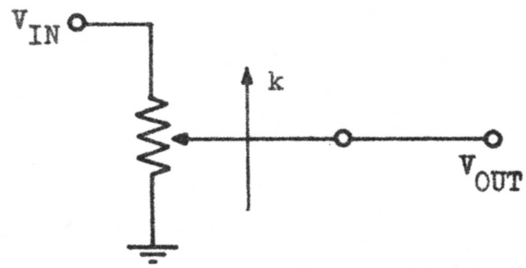


Fig. 2.6.

This might be done simply by its mechanical position, but since the potentiometer must be loaded in order to be used (i.e. some current must be drawn from the output terminal), it should be set by comparing input and output voltages with the load connected as described in Chapter 1. The symbol for a potentiometer so used is shown in Fig.2.7: the number of the potentiometer is placed inside the circle.

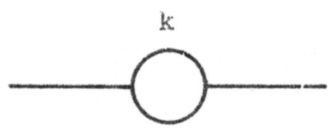


Fig. 2.7.

If 'k' lies between '1' and 10, a combination of potentiometer and a x 10 amplifier input will produce the required result, as in Fig. 2.8 where V_{IN} is multiplied effectively by 7.8.

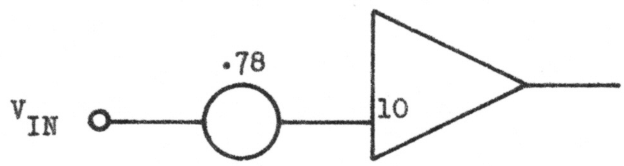


Fig. 2.8.

Alternatively, if a summing amplifier has only one input, or if all the several inputs to it have to be multiplied by the same constant and the constant is greater than one, the required result can be achieved by multiplying by the reciprocal of the constant in the feedback circuit.

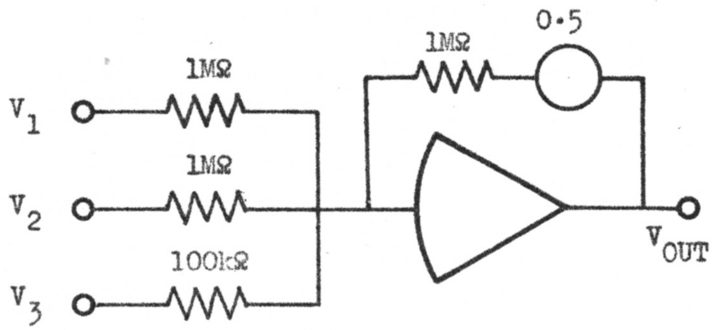


Fig. 2.9.

For example the circuit of Fig. 2.9 gives

$$V_{OUT} = -2(V_1 + V_2 + 10V_3)$$

Obviously care must be taken to see that the output voltage is not driven beyond 10V.

2.3. The Operational Amplifier: Integration.

2.3.1. The basic integrator.

It has been seen that, with one input voltage signal (assumed constant and positive for the moment) and a given input resistance, a certain constant current will be drawn from the signal source and will flow to the summing junction and thence along the feedback path. This must occur regardless of what is connected in the feed-back path, if the virtual earth point is to remain close to earth potential. Now, if the feedback path contains a capacitor, as in Fig. 2.10 it will be changed by the constant current, its charge will increase uniformly with time and so will the potential difference across it.

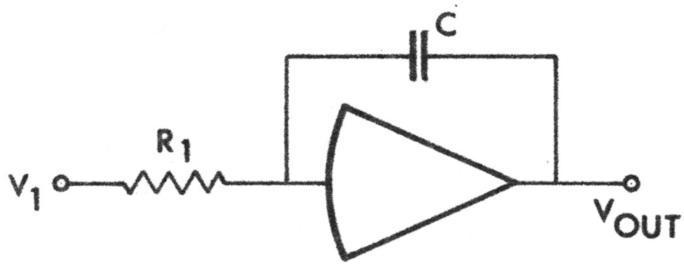


Fig. 2.10.

Thus, since the summing junction side of the capacitor is fixed virtually at earth potential, the output voltage will increase (negatively) at a constant rate.

Similar reasoning applies if the magnitude of the input voltage varies in proportion. Further, more than one input voltage may be applied and so we have

$$V_{OUT} = -\left[\int_0^t \frac{V_1}{CR_1}\,dt + \int_0^t \frac{V_2}{CR_2}\,dt \ldots\ldots\right]$$

The quantities CR_1 etc., have the dimensions of time and are known as "Time Constants", where it can be seen that the above relationship is dimensionally correct. In the Lan-Alog the time constants available are 1 sec., 0.1 sec. and 0.01 sec. Thus if, for instance, a time constant of 1 sec. is employed with an input of +1.0V, the output will become increasing negative at the rate of 1 volt in 1 sec: the smaller the time constant, the more rapid the change of output voltage. The symbol for an integrating amplifier is shown in Fig. 2.11.

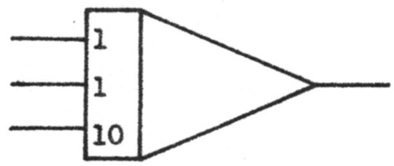


Fig. 2.11.

2.3.2. The Effect of the Computer Mode Controls.

The circuit of Fig. 2.10 applies when the Computer is in the "Compute" mode. In the "Reset" mode, the inputs are disconnected at the summing junction and two additional resistors are connected as in Fig. 2.12.

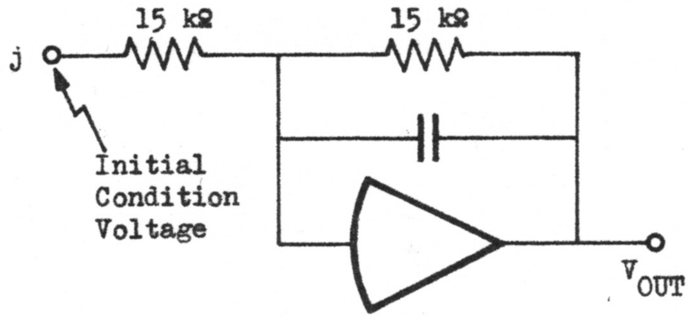


Fig. 2.12.

This circuit is effectively a single x 1 input summing amplifier with a capacitor connected across the feedback resistor. An initial condition voltage applied at the input terminal, which is coded 'j' on the Lan-Alog panel, appears, with a change of sign, at the output terminal and the capacitor is charged to this value before computing commences. Fig. 2.13 shows the Computer Flow Diagram symbol for the initial condition circuitry. The potentiometer adjusts the voltage to the value required.

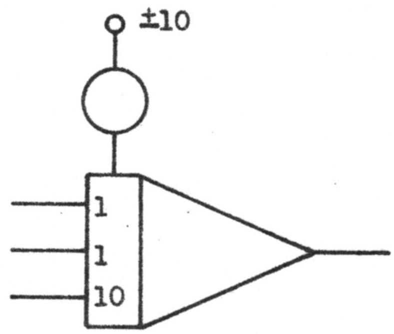


Fig. 2.13.

In the "Hold" mode, the summing junction is again disconnected. This, however, is the only alteration; its effect is simply to interrupt the current which charges the capacitor, the result being that the charge, and therefore the output voltage, are held at the values which obtained at the instant of switching to "Hold".

It should be noted that the amplifiers which are switched for use as summers are not effected by the mode switch.

2.4. Differentiation

Differentiation can be achieved by using capacitive input circuits with resistive feedback or - theoretically - by resistive inputs and inductive feedback. In practice differentiation is avoided because it seriously affects the quality of the signal if electrical noise (i.e. spurious voltage) is present. Equations are rearranged in such a way as to avoid the necessity of differentiation; this can normally be done without undue difficulty.

2.5 The First Order Linear Differential Equation.

The first order differential equation has the standard form

$$\tau \frac{dy}{dt} + y = Y$$

where τ (tau) is the time constant of the system described by the equation,

y is the variable quantity and
Y is the disturbance applied to the system.

For the purposes of this discussion, τ is assumed to be positive and Y is assumed to be a constant, in which case (and given zero initial conditions) Y is also the final value of y.

This is a simple differential equation with an equally simple and well known analytical solution which is that

$$y = Y(1-e^{\frac{-t}{\tau}})$$

for y = o when t = o. The variation of y with time is shown in Fig. 2.14 from which it can be seen that,

(i) y would have reached Y in a time τ had its initial rate of rise been maintained.

(ii) y actually rises to 63% of Y in time τ.

(iii) although theoretically y never reaches Y, the two are sensibly equal (better than 2%) after a time 4 τ.

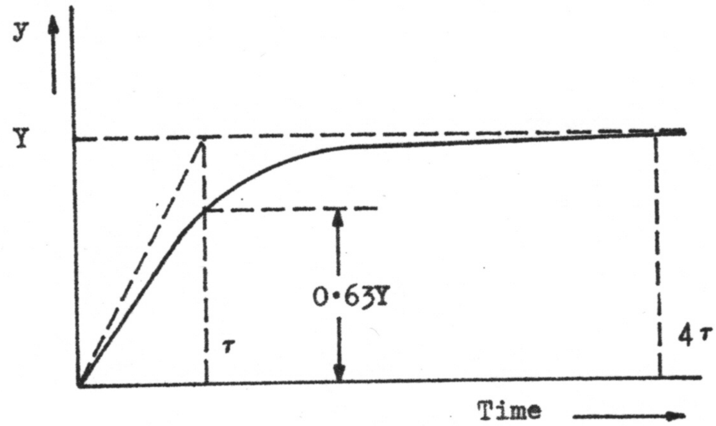


Fig. 2.14.

Before studying the application of an analogue computer to such an equation, it is as well to consider some of the physical systems which obey it.

(1) The Cistern.

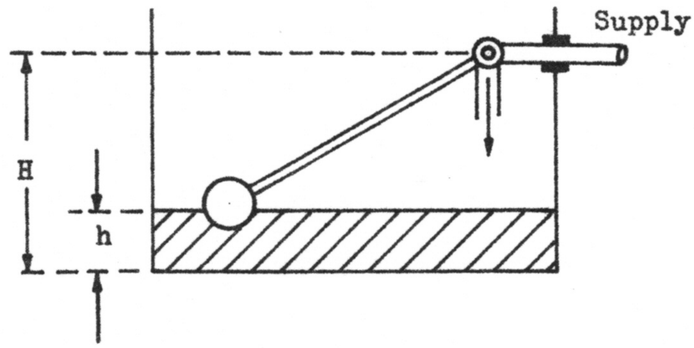


Fig. 2.15.

Suppose the rate of flow is proportional to the difference in levels (H - h). This is clearly an approximation for most practical cisterns, but it is nevertheless useful since it is easy to visualise.

We have

$$q = k\,(H - h)$$

The rate of change of level is given by

$$\frac{dh}{dt} = \frac{q}{A}$$

Combining

$$\frac{dh}{dt} = \frac{k}{A}\,(H - h)$$

or

$$\frac{A}{k}\frac{dh}{dt} + h = H$$

q in cu. ft/min.

k in cu. ft/min/ft. difference in levels

A tank area, sq. ft.

(2) Linear Acceleration of Body with Friction Proportional to Velocity.

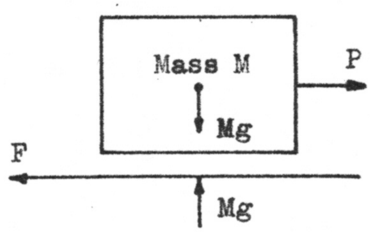


Fig. 2.16.

The horizontal forces on the body are a constant force P lbf and a frictional force F proportional to velocity, the constant of proportionality being k lbf/(ft/sec.).
If the velocity to the right is v ft/sec. then we have

$$M \frac{dv}{dt} = P - kv$$

or

$$\frac{M}{k}\frac{dv}{dt} + v = \frac{P}{k}$$

It is worth noting that this applies equally to the problem of a falling body, if wind resistance is assumed proportional to velocity and if the weight Mg, acting downwards, is substituted for P.

(3) An Electrical Circuit containing Inductance & Resistances.

The application of Kirchhoff's second law to the circuit in Fig. 2.17 gives the equation

$$L \frac{di}{dt} + Ri = E$$

or

$$\frac{L}{R}\frac{di}{dt} + i = \frac{E}{R}$$

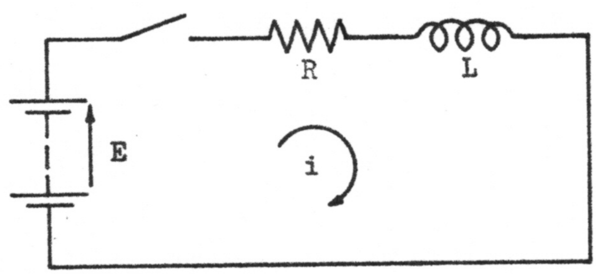


Fig. 2.17.

A circuit containing capacitance and resistance may be treated similarly using 'q' (charge) as the variable.

(4) Speed Control.

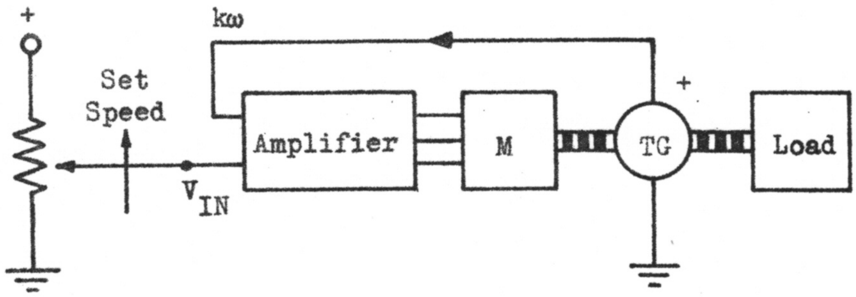


Fig. 2.18.

This in an example of a simple closed loop control system. The potentiometer slider is set according to the speed required and the slider voltage V_{IN} fed to the amplifier. The tacho-generator produces a voltage proportional to speed (kω) which is also fed to the amplifier. The amplifier motor combination produces a voltage proportional to the difference of its input voltages, the constant of proportionality being K. The effective moment of inertia of motor, tacho-generator and load is J. Friction is neglected and so

$$K\,(V_{IN} - k\omega) = J\; d\omega/dt$$

or

$$\frac{J}{Kk}\,\frac{d\omega}{dt} + \omega = \frac{V_{IN}}{Kk}$$

(5) Radio-Active Decay.

The rate of decay of radio active materials is proportional to the number of atoms of the material present. Let the disintegration constant be λ (lambda), then

$$-\delta N = \lambda N\,\delta t$$

The minus sign indicates a reduction in the number of atoms of the original substance. Whence we have

$$\frac{1}{\lambda}\,\frac{dN}{dt} + N = 0$$

This is a slight variation in that the final value of the variable is zero: therefore in order for the problem to have any meaning there must be a non-zero initial condition. If $N = N_o$ when $t = o$ the analytical solution is

$$N = N_o e^{-\lambda t}$$

and the graph is that of Fig. 2.19. This is an "upside down" version of Fig. 2.14 since the final value (zero) is necessarily smaller than the initial value.

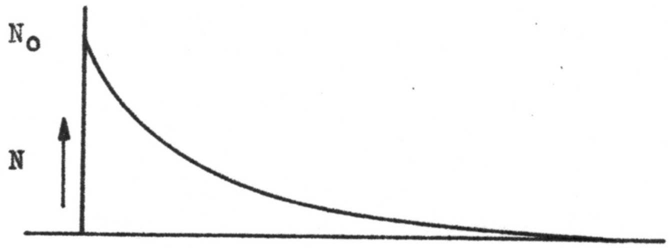


Fig. 2.19.

It will be seen that in each of the above illustrations the shape of the equation is

(Time Constant) x (Rate of Change of variable) + The variable itself = Final value of variable

The subjects of Programming and Scaling are dealt with in detail in Chapter 3 so it will suffice at this stage simply to give the computer Flow Diagrams in outline and to indicate briefly how these are constructed: it will be assumed that basically the problem is concerned with the differential equations for which the Analogue Computer is ideally suited.

To construct the Flow Diagram it is assumed that a voltage representing the highest derivative of a variable is available. This signal is arranged to be the input to a chain of integrators connected in series, the number corresponding to the order of the derivative. Voltages are now available to represent <u>all</u> the derivatives. These voltages are now combined in the way indicated by the problem equations, by summing amplifiers for instance or, in more advanced work, in multipliers and function generators.

Invertors may be required to get the correct sense at certain points. Amongst other things, these combinations will provide the highest derivation signal that was originally assumed: this is fed back to the input of the first integrator. Potentiometers are inserted where necessary for coefficients and scaling, as are initial condition circuitry and monitoring points.

If two or more simultaneous differential equations are to be solved, then there will be an equal number of chains of integrators and the voltages which are combined in a summing amplifier for instance may well be drawn from different integrator chains.

In the single first order equation with which we are dealing, there is only one chain and only one integrator in that chain-if such it can be called. Two possible arrangements are shown in Fig. 2.20. It will be seen that in (a) a single voltage representing $\frac{dy}{dt}$ is available and that one potentiometer alone is required to alter . These advantages are gained at the expense of using two additional amplifiers, one to do the summation and one to produce the correct sign at the integrator input. In practice economy in amplifiers is an over-riding consideration and, unless the $\frac{dy}{dt}$ signal were essential, the configuration of (b) would be used.

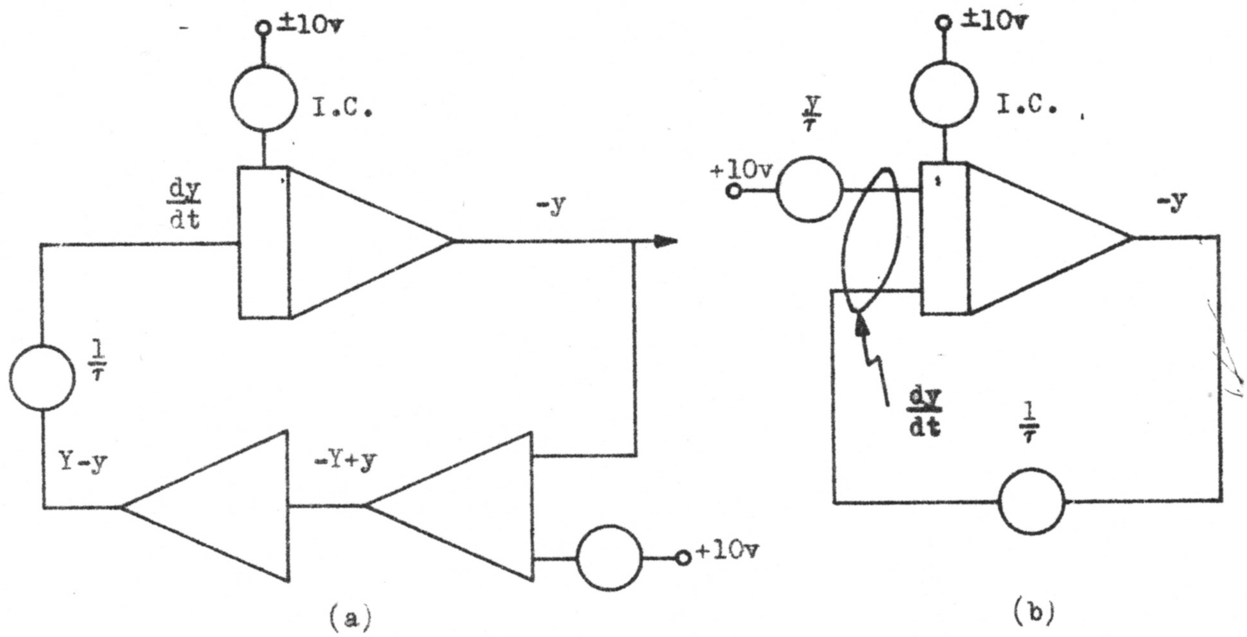


Fig. 2.20.

It should be noted that if what might be called the "frictional" quantity in the first order systems is made zero, the term in "y" disappears from the equation, which then reduces to

$$\frac{dy}{dt} = \text{a constant.}$$

To solve this, the feedback loop of Fig. 2.20(b) is omitted. (Fig. 2.20(a) is obviously superfluous). The resulting solution (the ramp function of Fig. 2.21) is useful as a slow time base for an oscilloscope or an disturbing function for a feedback control system.

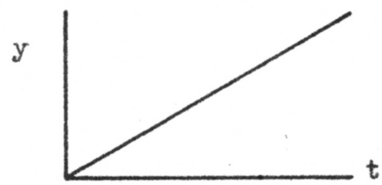


Fig. 2.21.

Our further point: the quantity Y of this section might not be a constant - it might for instance vary sinusoidally. This would imply a sinusoidally varying force in example (2) or an alternating supply voltage in (3) for instance. This would be simulated by using an oscillator, set to the required frequency, at the Y input instead of the computer reference supply.

2.6. The Second Order Linear Differential Equation

This equation describes the variations which occur in a simple oscillatory system. It is of fundamental importance in engineering and one standard form of it is

$$\frac{1}{\omega_o^2}\frac{d^2y}{dt^2} + \frac{2\zeta}{\omega_o}\frac{dy}{dt} + y = Y$$

where

y and Y are as previously defined

ω_o is the undamped natural frequency of the system and

ζ (zeta) is the damping ratio, the meaning of which is explained below.

In the absence of any disturbing function and with no damping the equation reduces to

$$\frac{d^2y}{dt^2} = -\omega_o^2 y$$

which will be recognised as the basic equation defining Simple Harmonic Motion and solution of which is

$$y = A \sin (\omega_o t + \phi)$$

A and ϕ (phi) being constants depending on the initial conditions.

If a term in $\frac{dy}{dt}$ is introduced (this is known as damping and would be for instance a frictional force proportional to velocity in a mechanical system or a resistance in an electrical circuit), the effect is to reduce the natural frequency and to cause the oscillations to decay exponentially. If the damping is high, the variations (in response to a step input Y = a constant, say) become entirely non-oscillatory and consists mathematically of two negative exponential terms alone. The boundary between oscillatory and non- oscillatory modes is the interesting condition known as "critical damping". The ratio

$$\frac{\text{(actual damping present)}}{\text{critical damping}}$$

is known as the "damping ratio".

The response of a second order system to a step input (Fig. 2.22) for various amounts of damping are given in Fig. 2.23 both graphically and mathematically.

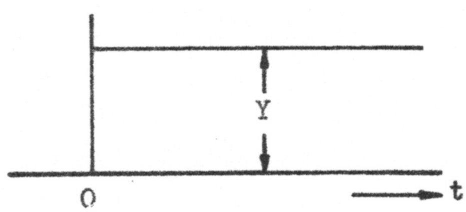


Fig. 2.22.

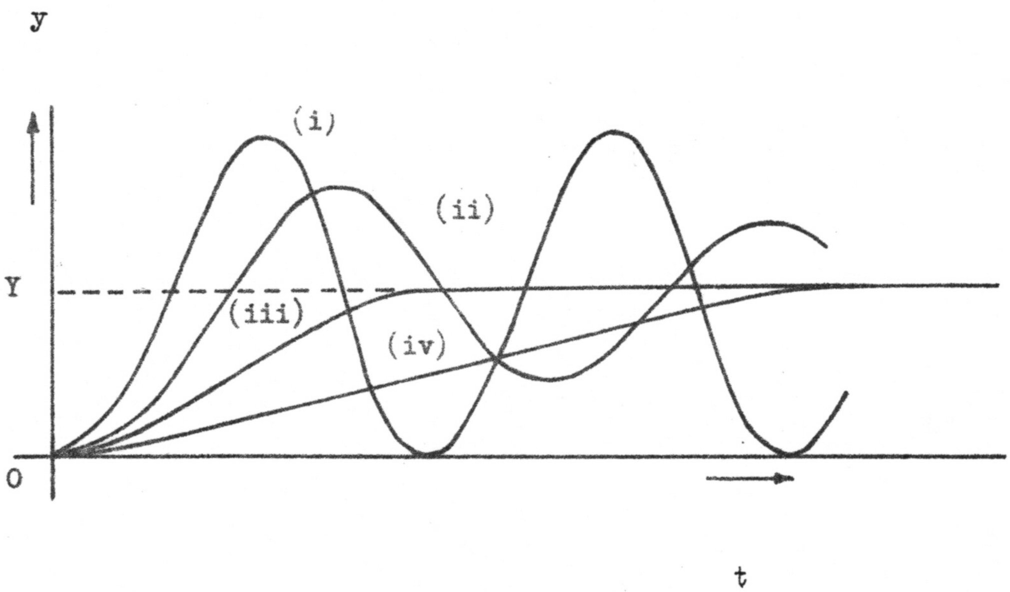


Fig. 2.23.

(i) $\zeta = 0$ (no damping)

$$y = Y(1 - \cos \omega_0 t)$$

(ii) $\zeta < 1$ (underdamped)

$$y = Y\left[1 - e^{-\zeta\omega_0 t} \cdot \frac{1}{\sqrt{1-\zeta^2}} \cdot \sin\left\{\sqrt{1-\zeta^2}\,\omega_0 t + \cos^{-1}\zeta\right\}\right]$$

(it will be seen that this formula reduces to (i) as $\zeta \longrightarrow 0$)

(iii) $\zeta = 1$ (critical damping)

$$y = Y\left[1 - e^{-\omega_0 t}(1 + \omega_0 t)\right]$$

(iv) $\zeta > 1$ (overdamped)

$$y = Y\left[1 - e^{-\zeta\omega_0 t} \cdot \frac{1}{\sqrt{\zeta^2 - 1}} \cdot \sinh\left\{\sqrt{\zeta^2 - 1}\,\omega_0 t + \cos^{-1}\zeta\right\}\right]$$

The derivation of the differential equations for three second order systems is given below.

(1) A mechanical system.

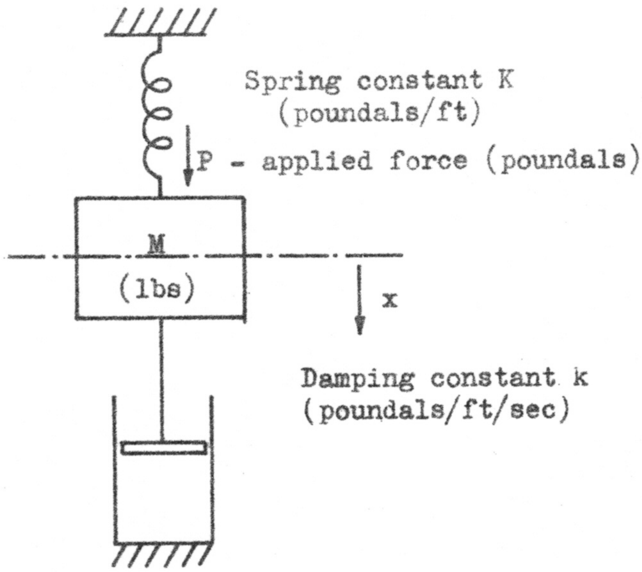


Fig. 2.24.

The displacement, denoted by x in Fig. 2.24, is measured from the static equilibrium position with P = 0 so that the weight need not be considered.

The spring force and damping force act upwards on the mass when x and $\frac{dx}{dt}$ are positive.

Hence $$P - k\frac{dx}{dt} - Kx = M\frac{d^2x}{dt^2}$$

or $$\frac{M}{K}\frac{d^2x}{dt^2} + \frac{k}{K}\frac{dx}{dt} + x = \frac{P}{K}$$

Comparison with the standard form of the equation shows that the undamped natural frequency $\omega_o = \sqrt{\frac{K}{M}}$ and that critical damping, for which $\zeta = 1$, is given by $k_{crit} = 2\sqrt{MK}$

(2) An electrical circuit.

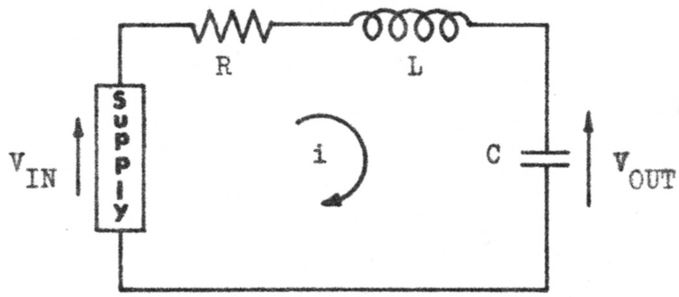


Fig. 2.25.

Applying Kirchoff's second law we have

$$V_{IN} = L\frac{di}{dt} + Ri + \frac{q}{c}$$

Substituting for i and multiplying by C gives

$$LC\frac{d^2q}{dt^2} + CR\frac{dq}{dt} + q = CV_{IN}$$

whence

$$\omega_o = \frac{1}{\sqrt{LC}}$$

and

$$R_{crit} = 2\sqrt{\frac{L}{C}}$$

(3) A position control servomechanism.

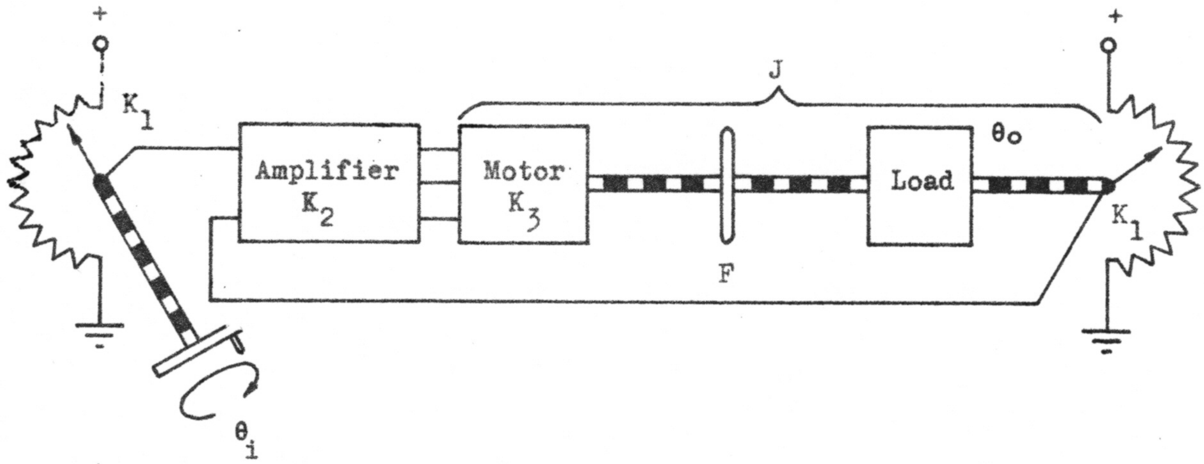


Fig. 2.26.

In Fig. 2.26 the demanded position (θ_i) and the actual position (θ_o) are compared by the potentiometer arrangement: the amplifier and motor drive the system until the two are in agreement.

Let J be the effective moment of inertia of motor and load (kilogram - metre^2)

F be the damping torque coefficient (newton - metre/radian/sec)

K_1 be potentiometer sensitivity (volt/radian)

K_2 be amplifier gain (amp/volt)

K_3 be motor torque constant (newton - metre/amp.)

where

$$K_1 K_2 K_3 = K = \text{stiffness of system}$$
$$= \text{torque/unit error}$$
$$= \text{newton metre/radian}$$

We have

$$K(\theta_i - \theta_o) - F\frac{d\theta_o}{dt} = J\frac{d^2\theta_o}{dt^2}$$

or

$$\frac{J}{K}\frac{d^2\theta_o}{dt^2} + \frac{F}{K}\frac{d\theta_o}{dt} + \theta_o = \theta_i$$

giving

$$\omega_o = \sqrt{\frac{K}{J}}$$

and

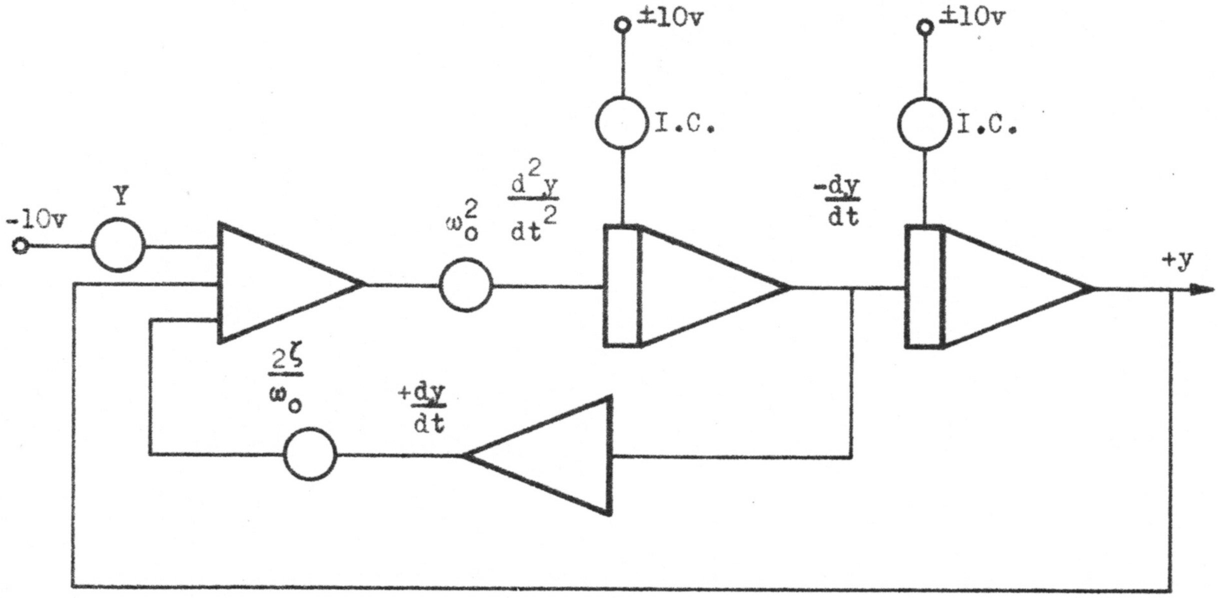
$$F_{crit} = \sqrt{2JK}$$

The Computer Flow Diagram is drawn by the procedure outlined previously. The equation to be solved is

$$\frac{d^2y}{dt^2} = \omega_o^2\left(Y - y - \frac{2\zeta}{\omega_o}\frac{dy}{dt}\right)$$

Three or four amplifiers may be used, the consideration of convenience versus economy of equipment being similar. The diagrams are given in Fig. 27.

Fig. 2.27 (a)



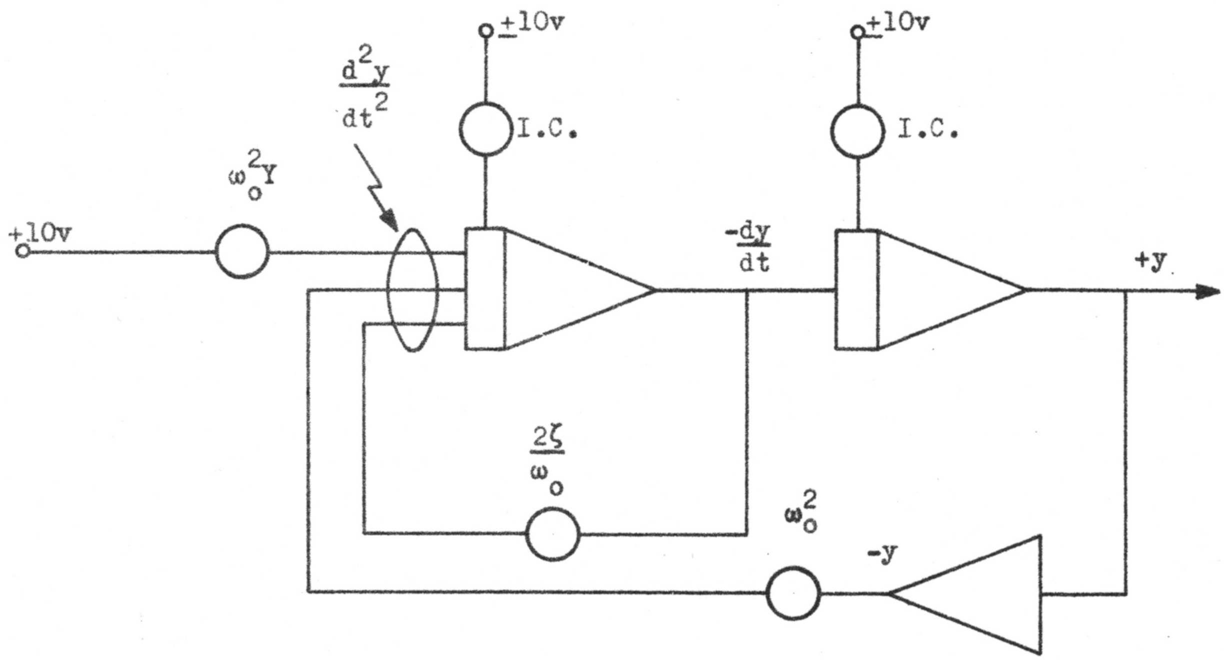


Fig. 2.27 (b)

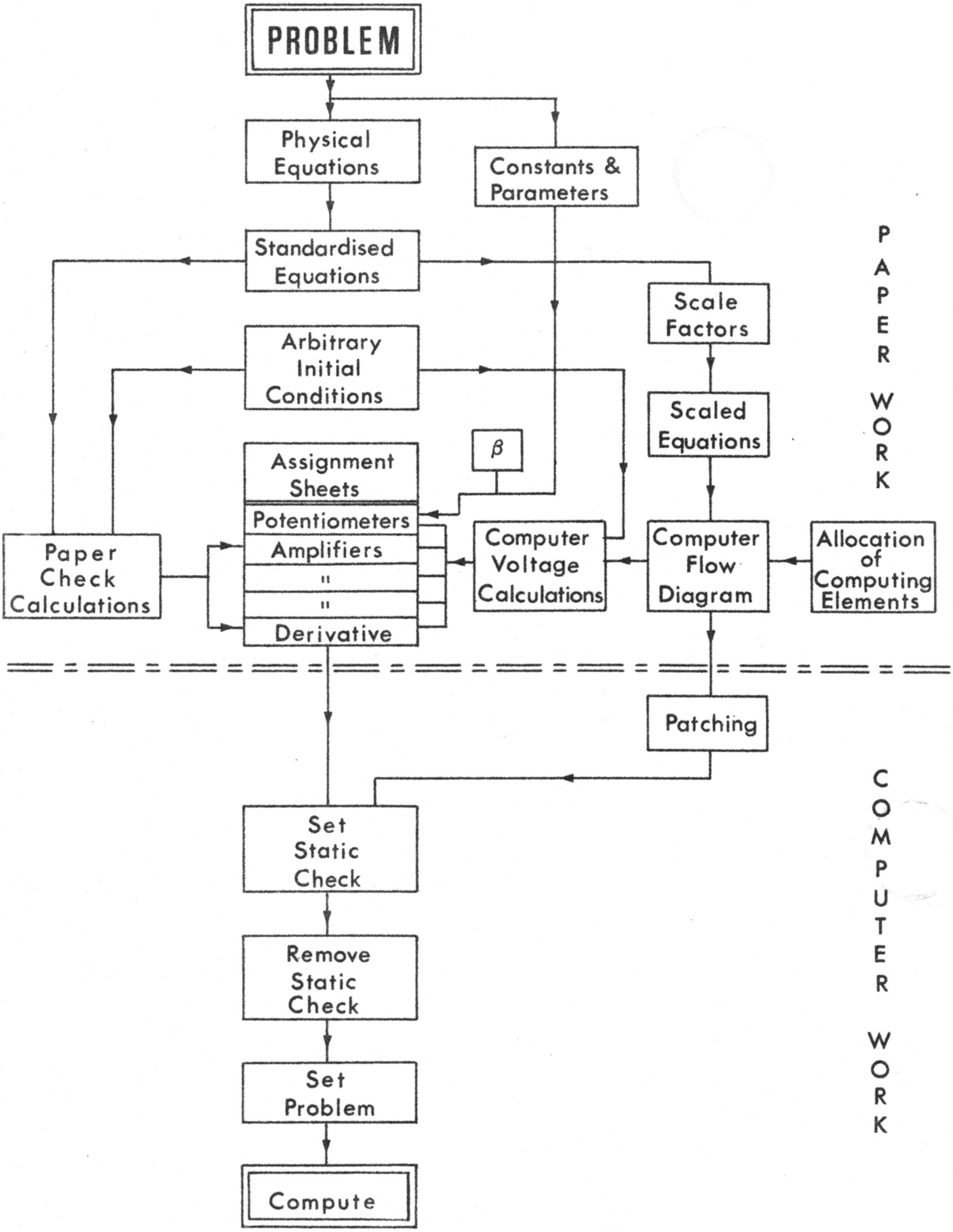

PROBLEM
Physical Equations
Constants & Parameters
Standardised Equations
Scale Factors
Arbitrary Initial Conditions
Scaled Equations
β
Assignment Sheets
Potentiometers
Amplifiers
"
"
Derivative
Paper Check Calculations
Computer Voltage Calculations
Computer Flow Diagram
Allocation of Computing Elements
PAPER WORK
Patching
Set Static Check
Remove Static Check
Set Problem
Compute
COMPUTER WORK

Fig 3·1

CHAPTER 3.

PROGRAMMING AND SCALING

3.1. Introduction.

The treatment of Chapter 2 has been academic rather than practical: it is the purpose of this chapter to consider the programming of actual problems, complete with numerical data, on the Lan-Alog computer. It may seem that some of the procedures outlined are trivial or pedantic when applied to the comparatively simple problems so far described, but it is the writer's belief that a logical and systematic method is of the greatest value in tackling problems of any complexity, and that such a method should be inculcated from the beginning. It becomes essential if more than one person is involved in the solution. The reader is advised to study one of the worked examples, Section 4.4 for instance, in conjunction with this chapter.

3.2. Programming

The standard procedure is shown diagrammatically in Fig. 3.1. The object is to ensure that the computer circuitry is arranged correctly to solve the original problem and that no errors of calculation, of patching, or due to faulty equipment remain undetected.

It will be seen that there are two independent paths of working, starting at the Standardised Equations and linked by the Assignment Sheets. This is a self-checking feature, provided that no cross-references are made from one path to the other -in particular that the Paper Check calculations are made from the Standardised Equations and never from the Scaled Equations. This ensures that paperwork errors are removed before going on to the computer, but it should be noted that the steps leading to the Standardised Equations are not checked and must therefore be done with especial care. This applies also to the Constants and the Initial Conditions.

3.2.1. Programming-Paper Work.

(a) Having studied the Problem and formed the Physical Equations by the normal methods, some modifications are usually necessary to make the equations amenable to programming, the results of which will be called the Standardised Equations. Firstly, all **differential** equations of order greater than one are re-written so that each is of first order.

Thus there will be as many first order equations as the order of the original equation and some extra symbols will be necessary. The differential coefficient term is placed on the left of the equality and all other terms and factors on the right. Each first order equation will be associated with a particular integrating amplifier and so the general shape may be written

$$\frac{d}{dt}\ \text{of the output variable of a particular integrator} = \text{the sum of a series of terms obtained from the Physical Equations, each of which will appear at the integrator input}$$

Secondly the output of every other amplifier (except a 1:1 inverter) is given a symbol and the corresponding equation written down. These outputs may be from summers or, in more advanced computing, from multipliers or function generators. The necessary substitutions are made in the first order differential equations mentioned above.

(b) The Standardised Equations are next scaled as described in 3.4. below.

(c) Initial Conditions are values of problem variables which obtain when t = o. They are applied to the computer as voltages which are set on the feedback capacitors of integrating amplifiers in the Reset mode. How many such voltages have to be set depends on the requirements of the problem, but, before a computation can be attempted, the programming and the computer itself must be subjected to a comprehensive Static Check. For this purpose values must be chosen for

(i) the initial conditions of <u>all</u> integrators.

(ii) any other quality (such as a problem constant or a disturbing function value) which is derived directly from the Computer Reference supply or any other voltage source.

These values are arbitrary except that they must not be zero and they must not result in voltages which exceed the Computer Reference Voltage (10V).

(d) The Computer Flow Diagram is now drawn completely-parts of it may well have been sketched when producing the Standardised Equations. The symbols representing potentiometer, amplifiers and other equipment are marked with reference numbers as they are allocated. The Diagram is labelled with the <u>scaled</u> variables (with voltages, that is) and above the potentiometer symbols are written the mathematical symbols for the relevant coefficients.

(e) Assignment Sheets, such as those used in the worked examples, are now compiled. Apart from listing the potentiometers, amplifiers etc., and the function each performs, these sheets will contain the output voltage of each element for various problem conditions, the first condition being normally the Static Check. These voltages are calculated from the Scaled Equations which it should be realised are voltage equations. Each term in each Scaled Equation should correspond to a part of the Computer Flow Diagram and it is convenient to check the Diagram as these calculations proceed. It is advisable to deal with these equations in the order in which they can be solved directly from the data, so that mistakes are propagated downwards only.

The scaled equations may be classified in two groups as were the standardised equations; those associated with integrators and those associated with summers (or with multipliers, function generators etc). The results of the voltage calculations give the output voltages of the summing (and other) amplifiers, but they give the input of the integrating amplifiers; which for obvious reasons are called derivatives. (The outputs of the integrators; in the Reset mode when the Static Check is performed; are simply the initial condition voltages). The summing (and other) amplifier outputs are entered on the amplifier sheet, while the derivatives are tabulated on a "Derivative Sheet" against the corresponding integrating amplifier numbers.

(f) The Paper Check. The Initial Conditions are substituted in the Standardised Equations - also in the order in which the equations can be solved directly.

This step results in physical quantities being calculated for the left hand side of each Standardised Equation. These physical quantities are compared with

(i) the Derivative Check Sheet voltages and

(ii) the output voltages, on the Amplifier Sheet, of those amplifiers which are summers (or associated with non-linear equipment, if any),

taking into account the scaling indicated on the sheets.

Agreement makes it reasonably certain that the paper work is numerically correct. The Problem is now ready for the Computer.

3.2.2. Programming - On the Computer

This is comparatively straight forward and consists of the following steps which are carried out with the computer in the "Reset" mode:-

(a) The connections are patched together according to the Computer Flow Diagram. It is usual to ensure that all connections are made by drawing the diagram originally in pencil and inking it in as patching proceeds. In the case of the worked examples in this handbook the connections are coded, in the way described in Chapter 1, and listed.

(b) The potentiometer coefficients are adjusted to the values indicated in the first of the setting columns on the potentiometer sheet: this is the column normally used for the comprehensive Static Check.

(c) All amplifier outputs are checked against the Amplifier Sheet.

(d) All integrator inputs are checked against the Derivative Sheet. To do this, the link 'h-m' is disconnected from the integrator in question and 'h' is patched to the base socket 'm' of a spare amplifier. The check is then made at the output of the spare amplifier.

Care must be taken in 'b' and 'd' to return the patching to its original state after each potentiometer or integrator has been dealt with.

Consideration will show that once this stage has been reached successfully it is known that the problem has been programmed correctly and that every relevant computer circuit except one is functioning properly. The exception is the feedback capacitor of each integrating amplifier. Should the integrating function be suspect, a separate rate test may be applied e.g. if 1.0V is applied to a x1 input, then the output voltage should change at the rate of 1.0V per second when switched to "Compute".

3.3. The Problem

The Static Check values are removed and replaced by those required for the problem. This means essentially that the potentiometer coefficients are adjusted to the values indicated in the second and subsequent settings columns of the Potentiometer sheet. The amplifier outputs are checked against the relevant column of the Amplifier Sheet. Those variables which it is required to monitor are connected to oscilloscope or pen recorder and when all is ready the computer is switched to the "Compute" mode.

3.4. Scaling

There are two aspects of scaling and several methods of tackling each of them. Both Amplitude and Time Scaling will be dealt with, but only one method - that which is most commonly advocated and used,- will be considered in order to avoid confusion. Others may be studied in the references.

3.4.1. Amplitude Scaling

The Standardised Equations are used. A list is made of the output variables of all amplifiers: consideration will show that this list will correspond to (1) the left hand side of those Standardised Equations which relate to summing amplifiers and (2) the quantities which are operated on by $\frac{d}{dt}$ on the left hand side of those Standardised Equations which relate to integrators. Beside the quantities so listed are entered their estimated maximum values. Some insight into the system being investigated may well be required for this.

From the variable and its maximum value, together with a knowledge of the maximum value of the computing voltage (10V in the case of the Lan-Alog Computer), is derived the "Scaled Variable" which is

$$\left(\frac{\text{variable quantity}}{\text{its maximum value}} \times \text{ the Computing Voltage}\right)$$

For instance, consider scaling the velocity of a linear mechanical vibrating system, which has a maximum displacement of 2 feet and a maximum resonant frequency of 6 c/s. Then

$$v_{max} = 2\pi \times 6 \times 2 \text{ ft/sec}$$

$$= 75.5 \text{ ft/sec}$$

The figure 75.5 would be rounded off to 80 and for the Lan-Alog the scaled variable would be

$$\left(\frac{v}{80} \times 10\right) \text{ i.e. } \left(\frac{v}{8}\right)$$

The scaled variables have the dimensions of voltage and must be thought of as such. Furthermore <u>the brackets are retained and their contents never altered</u> in subsequent work - unless of course a change in the scaling is found to be necessary.

The Scaled Variables are now substituted for the physical variables in the Standardised Equations and each term on the right hand side of the resultant "Scaled Equation" is multiplied by whatever extra factors are necessary to make the Scaled Equation agree numerically with the Standardised Equation.

In each term in each Scaled Equation, all the quantities outside the Scaled Variable brackets (that is the original factors in the Standardised Equations and the extra factors introduced by scaling) are multiplied together to give a coefficient which will be set on a potentiometer.

3.4.2. Time Scaling

Computer solutions normally occupy a few seconds. The time required for the physical system to undergo the corresponding variations may vary from micro-micro-seconds (in a high frequency electrical circuit) to years (in an economic system). The effect of this vast disparity is that, for a system which does not "fit" the natural time of solution of the computer, the coefficients required to be set lie outside the range which can be achieved conveniently by combination of potentiometers and x1 or x10 amplifier inputs, that is between say 0.01 and 10.

To overcome this difficulty, the physical system is imagined to be slowed down or speeded up by factor β and Computer Time τ (tau) is introduced, where $\tau = \beta t$. If $\beta > 1$, the system, or rather its analogue, is slowed down i.e. it is made to take longer over its variations.

Since $\tau = \beta t$,

$$\frac{d}{d\tau} = \frac{d}{d(\beta t)} = \frac{1}{\beta}\frac{d}{dt}$$

and

$$\frac{d^2}{d\tau^2} = \frac{d^2}{d(\beta t)^2} = \frac{1}{\beta^2}\frac{d^2}{dt^2} = \frac{1}{\beta}\frac{d}{dt}\left(\frac{1}{\beta}\frac{d}{dt}\right) \quad \text{etc.}$$

Time Scaling affects only the equations related to integrators- obviously summation and multiplication, for instance, will not be affected. Analytically the effect is that the left hand side of each Scaled Equation becomes

$$\frac{d}{d\tau}\left(\begin{matrix}\text{Scaled} \\ \text{variable}\end{matrix}\right)$$

and all the terms on the right hand sides are multiplied by $1/\beta$. This simple modification is made a standard procedure and a value for β chosen by inspection when all other factors are known, the object being to make all the net products lie between 0.01 and 10.

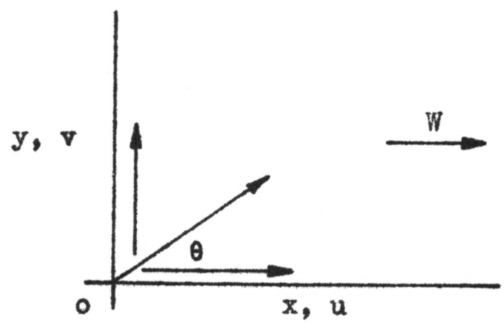
Consideration of the shape of the Standardised Equations will show that, where a higher order (say third) derivative is integrated successively to produce the variable itself, the necessary power of $1/\beta$, that is $(1/\beta)^3$ is produced by a series of three factors of $1/\beta$ affecting the input to each integrator in turn.

CHAPTER 4.

ILLUSTRATIVE EXAMPLES USING ONE LAN-ALOG COMPUTER

A PROJECTILE PROBLEM

(1) This is a 2- dimensional problem in simple kinematics. Wind resistance is assumed to be proportional to speed (it isn't, but non-linear equipment is required to cater for powers of the speed other than 1). The motions in the two axes are independent and may be solved singly.



Let 'm' be mass of projectile, 'f' be drag or resistance per unit speed relative to the air and 'W' be wind speed.

(2) Physical Equations

$$m \frac{d^2x}{dt^2} = - f \left(\frac{dx}{dt} - W\right) = fW - f \frac{dx}{dt}$$

$$m \frac{d^2y}{dt^2} = - mg - f \frac{dy}{dt}$$

(3) Standardised Equations

$$\frac{du}{dt} = \frac{fW}{m} - \frac{f}{m} u$$

$$\frac{dx}{dt} = u$$

$$\frac{dv}{dt} = - g - \frac{f}{m} v$$

$$\frac{dy}{dt} = v$$

(4) Scaling

Suppose the data is that the maximum initial velocity is 100 ft/sec at any angle θ from 0° to 90° to the horizontal.
(An initial calculation will be necessary to find the horizontal and vertical components).

W lies between 0 and $\pm$ 50 ft/sec
m = 1.6 lb (a football perhaps)
f = 0.05 lb f/(ft/sec)

Preliminary calculations; firstly it is advisable to use consistent units (i.e. units linked by 1 : 1 : 1 definition, which the lb mass, lb force and ft/sec^2 are not) so let us change say 1.6 lb to $1.6/_{32}$ i.e. $1/_{20}$ slug (since 1 lb f gives 1 slug an acceleration of 1 ft/sec^2).

Secondly, we need an estimate of the maximum distances involved.

The maximum time of flight given by

$$\frac{\text{200 ft/sec (max change of vertical velocity)}}{\text{32 ft/sec}^2 \text{ (g)}} = \text{6 second approximately.}$$

Max range is achieved with $\theta = 45^{\circ}$ approx and will be approximately $6 \times \frac{100}{\sqrt{2}}$, say 500 ft. We will use the same scale vertically and then the scaled variables are

$$\left(\frac{x}{500} \times 10\right),\ \left(\frac{y}{500} \times 10\right),\ \left(\frac{u}{100} \times 10\right) \text{ and } \left(\frac{v}{100} \times 10\right)$$

(5) The Scaled Equations

$$\frac{d}{d\tau}\left(\frac{u}{10}\right) = \frac{fW}{m}\cdot(10)\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{\beta} - \frac{f}{m}\cdot\left(\frac{u}{10}\right)\cdot\frac{1}{\beta}$$

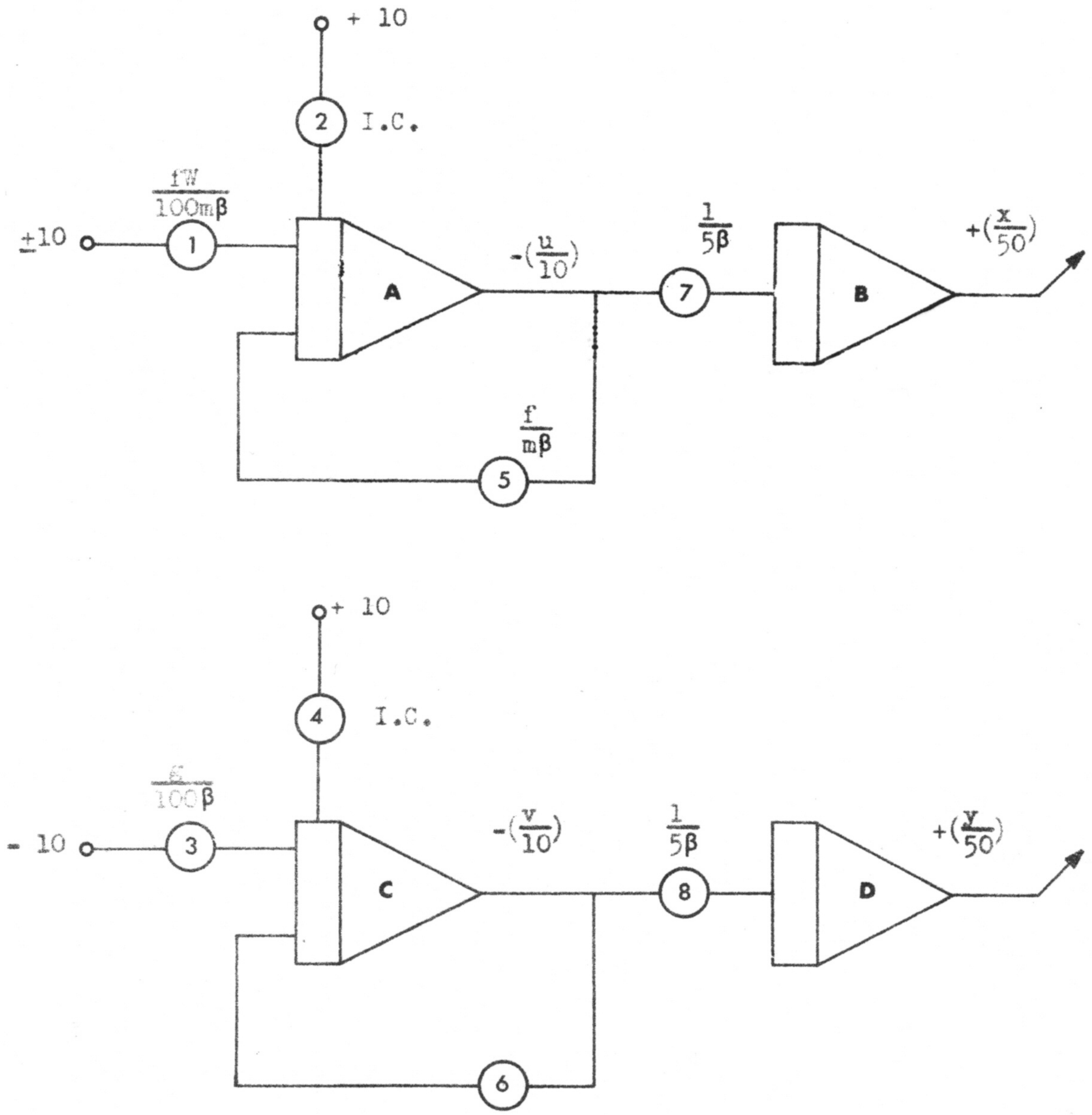
$$\frac{d}{d\tau}\left(\frac{x}{50}\right) = \left(\frac{u}{10}\right)\cdot\frac{1}{5}\cdot\frac{1}{\beta}$$

$$\frac{d}{d\tau}\left(\frac{v}{10}\right) = -\,g\,(10)\,\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{\beta} - \frac{f}{m}\left(\frac{v}{10}\right)\frac{1}{\beta}$$

$$\frac{d}{d\tau}\left(\frac{y}{50}\right) = \left(\frac{v}{10}\right)\frac{1}{5}\cdot\frac{1}{\beta}$$

(Projectile Problem)

(6) The Computer Flow Diagram



(Projectile Problem)

Evidently more equipment is required (8 potentiometers) than is available on one Lan-Alog. This frequently occurs in analogue computing and various subterfuges may be used to get round the problem. Thus we may:-

1) restrict the problem

e.g. (a) solve one motion at a time.

(b) omit wind and have equal initial conditions for u and v.

(c) omit wind and make $\beta = \frac{100}{32}$ so that pot 1 is superfluous.

(d) best, speed up the problem 5-fold ($\beta = \frac{1}{5}$) so that pots 7 and 8 are not required.

2) use extra equipment if available

e.g. (a) fixed resistors might be used for pot 2

(b) an external pot (or possibly fixed resistors) could be used for pot 1.

In what follows 1) (d) is used: the coefficients required then for the other potentiometers are still suitable.

(7) The Initial Conditions

let $W = 20$ ft/sec

$u = v = 30$ ft/sec

$x = y = 500$ ft, (so that no further potentiometers are required)

(8) The Assignment Sheets.

(a) Potentiometers

Ref. No.	Parameter	Static Check	Run 1	Run 2
1	$fW/100\ m\beta$	1.00		
3	$g/100\ \beta$	-0.16 x 10		
2	I.C. of $-(\frac{u}{10})$	+ 0.3		
4	I.C. of $-(\frac{v}{10})$	~~+ 0.3~~ -0.3		
5	$f/m\ \beta$	0.5 x 10		
6	$f/m\ \beta$	0.5 x 10		

(Projectile Problem)

(b) Amplifiers

Amplifier No.	Output	Static Check Voltage		
A	-(u/10)	-3.0		
C	-(v/10)	-3.0		
B	+(x/50)	10.0		
D	+(y/50)	10.0		

(c) Derivatives

Amplifier No.	Inputs	Total		
C	-10 x 0.16 x 10 -3 x 0.5 x 10	- 31V	Too large - make v =	- 30
C revised	-10 x 0.16 x 10 + 3 x 0.15 x 10	-1.00V		
A	+10 x 1.00 x 1 - 3 x 0.5 x 10	-5.00V		
B	- 3 x 1.00 x 1	-3.00V		
D	+ 3 x 1.00 x 1	+3.00V		

These will appear as + 1.00 + 5.00 + 3.00 and - 3.00 at the output of the amplifier used to check them. On the Lan-Alog Amplifier D would be used to check A, B & C and then amplifier C, to check D, taking care to return C to its original condition. (In this problem since nothing else is affected C and D may be checked more simply by switching to Σ).

(9) The Paper Check.

From the standardised equations we have

$$\frac{du}{dt} = \frac{fW}{m} - \frac{f}{m}u = \frac{0.05}{1/20} \times 20 - \frac{0.05}{1/20} \times 30 = -10 \text{ ft/sec}^2$$

$$\frac{dv}{dt} = -g - \frac{f}{m}v = -32 - \frac{0.05}{1/20}(-30) = -2\text{ft/sec}^2$$

$$\frac{dx}{dt} = u = +30 \text{ ft/sec}$$

$$\frac{dy}{dt} = v = -30 \text{ ft/sec}$$

Hence we have

Amplifier No.	Derivative	Value	Scale *	Voltage	Check with 8(c)
A	du/dt	-10	$+\frac{1}{\beta t}(\frac{u}{10})$	-5.00	
C	dv/dt	- 2	$+\frac{1}{\beta t}(\frac{v}{10})$	-1.00	
B	dx/dt	+30	$-\frac{1}{\beta t}(\frac{x}{50})$	-3.00	
D	dy/dt	-30	$-\frac{1}{\beta t}(\frac{y}{50})$	+3.00	

* In this column u, v, x, y and t are irrelevant numerically - they merely give the dimensions, as explained previously. Thus the voltage of amplifier 1 is calculated

$$(-10) \times \frac{1}{1/5} \times \left(\frac{1}{10}\right) = -5.00\text{V}$$

This programme may be used

(i) to plot the trajectory under various conditions

(ii) to find the maximum range vertically and horizontally (if the latter were more than 500 ft some rescaling would be required)

(iii) to find how a footballer might kick into a wind and have the ball return to his feet - or more entertainingly to see if a centre-forward could contrive to kick-off and score in his own goal because of a strong head wind.

THE PROJECTILE PROBLEM.

Switches:- All to $\int$

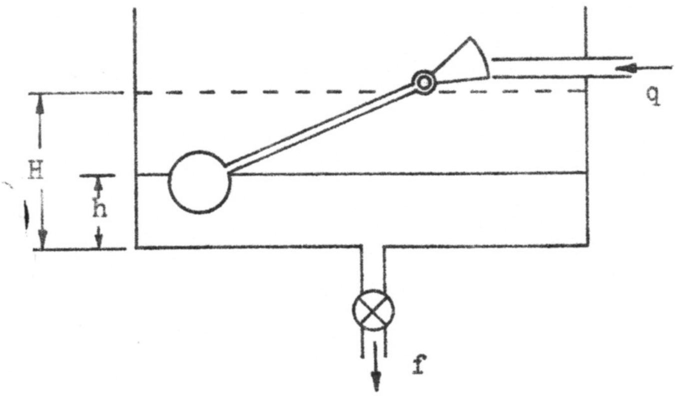
Horizontal Motion			Vertical Motion		
Aa(or Ab)	-	Ac	Cb	-	Cc
Ad-[0.1MΩ*]	-	Ae [or Ah*]	Cd	-	Cg
Ba	-	Bc	Da	-	Dc
Bd	-	Aj	Dd	-	Cj
Ar	-	As	Cr	-	Ds
Au	-	Av	Du	-	Dv
At	-	Ag	Dt - 0.1MΩ	-	Ch
Ar	-	Be	Cr	-	De
Br	-	Display	Dr	-	Display

Suitable for repetitive operation at about 1 c/s.

* if required (when W is large).

(1) The Cistern of Section 2.5 (1)

In order to relate the problem to real life, provision is made for an outflow.



Notation:-

H = height of water when flow in stops

h = instantaneous height of water

q = rate of flow in

K = flow constant (linearity and constancy assumed)

f = outflow

A = floor area.

Data:-

H = 3 ft

K = 4 cu ft/min per ft difference

f_{max} = 1 cu ft/sec

A = 10 sq ft

(2) The Physical Equation

We have assumed $q = K(H-h)$ and abviously

$dh/dt = (q-f)/A$. From these

$$\frac{A}{K}\frac{dh}{dt} + h = H - \frac{f}{K}$$

whence, note, the time constant of the system is $2\frac{1}{2}$ min.

(3) The Standardised Equation

This is

$$\frac{dh}{dt} = \frac{K}{A} H - \frac{K}{A} h - \frac{f}{A}$$

(4) Scaling

Amplifier output variable	Maximum value	Scaled variable (10V computer)
h	3	$(\frac{h}{3} \times 10)$
f *	1	$(\frac{f}{1} \times V)$

f^* is treated as a variable because the problem might well require it to vary with time in some manner. The voltage representing f would then be the output of a signal generator and V would be its value corresponding to f max.

So, scaling first for amplitude only, we have

$$\frac{d}{dt}\left(\frac{10h}{3}\right) = \frac{KH}{A}\,(10)^{\nabla}\,\overset{**}{\frac{1}{3}} - \frac{K}{A}\left(\frac{10h}{3}\right) - \frac{1}{A}\left(\frac{Vf}{1}\right)\overset{**}{\frac{10}{3V}}$$

$^{\nabla}$This is the 10V computer supply which is used for a constant such as H.

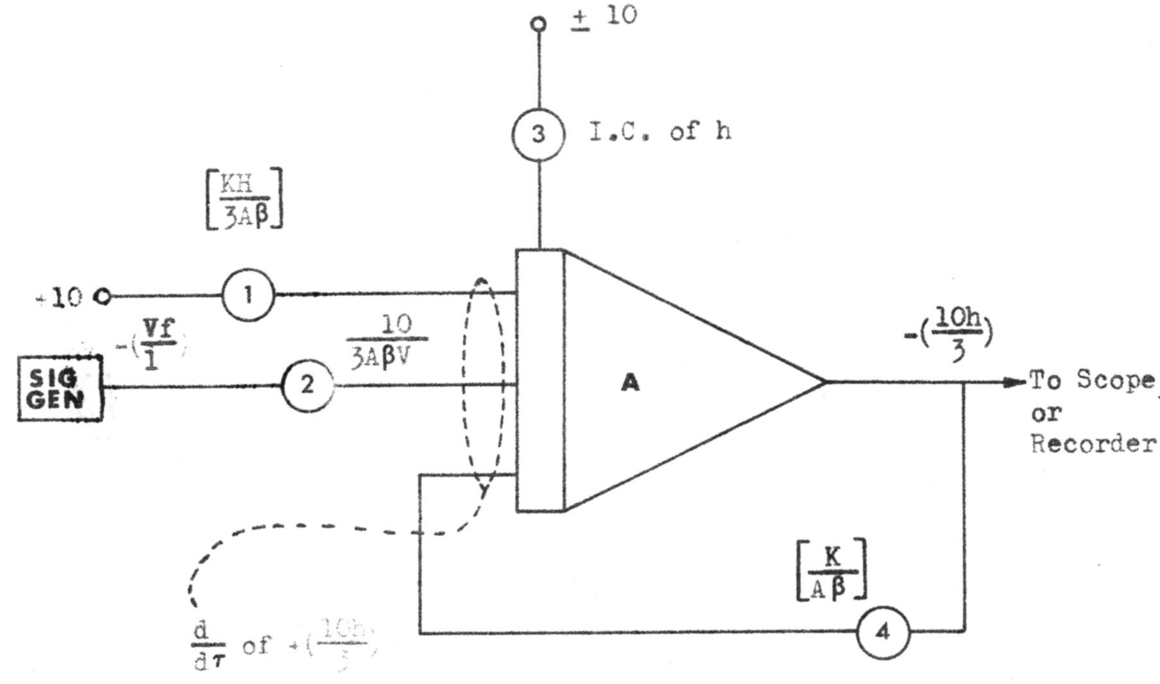
**These are the extra factors required to maintain the same equality between left hand side and right hand side as appeared in the Standardised Equation.

Time scaling results in

(5) The Scaled Equation

$$\frac{d}{d\tau}\left(\frac{10h}{3}\right) = \frac{KH}{A}\,(10)\,\frac{1}{3\beta} - \frac{K}{A}\left(\frac{10h}{3}\right)\frac{1}{\beta} - \frac{1}{A}\left(\frac{Vf}{1}\right)\frac{10}{3V\beta}$$

(6) The Computer Flow Diagram



(7) The Initial Conditions

There is only one integrator; its output is $-(\frac{10h}{3})$. Let the tank be half full i.e. h = 1.5 ft. Then the output voltage will be -5V and the input +5V since in the Reset mode, when Initial Conditions are set, the integrator is effectively a 1:1 inverter. Thus pot 3 will be set to 0.5, using the +10V supply.

°Had f been a constant it would have been treated like H and this term would have been $-\frac{f}{A}(10)\frac{1}{3\beta}$ This will be used in the static check.

f must be chosen with care as it can be large compared with q max and this might result in voltages greater than 10V. f = 0.1 is used below.

(8) The Assignment Sheets

(a) Potentiometers

Ref. No.	Parameter	Settings Static Check	Run 1	Run 2	Run 3
1	$KH/3A\beta$	0.40	0.40	0.40	0.40
2	$f/3A\beta$	0.20	0	0.30	1.4 (0.14 x 10
3	I.C. of Int 1	~~0.50~~ 0.25	0	1.00	1.00
4	$K/A\beta$	0.40	0.40	0.40	0.40

Inserting known numerical values gives the parameters as $1/150\beta$, $1/300\beta$ and $1/150\beta$ (pots 1, 2 and 4) and inspection shows that β must lie between $1/1.5$ and $1/3000$ if the settings are to be between 0.01 and 10. If $\beta = 1/60$ minutes of real time become seconds of computer time. This value has been used for the settings above.

Run 1 is for tank initially empty and no outflow

Run 2 is for tank initially full and an outflow of 0.15 cu ft/sec

Run 3 is for tank initially full but outflow = 0.7 cu ft/sec. This will empty the tank in due course - in fact the h voltage will become + ve indicating a water level below the bottom of the tank. The run could be used to find out how long such an outflow can be sustained without h falling below say 6" (-1.67V)

(b) Amplifiers

Amplifier No.	Output	Static Check Voltage
A	$-(\frac{10h}{3})$	- 2.5

(c) Derivatives

Amplifier No.	Inputs	Total
A	+10V x 0.4 x 1 = +4.0 -10V x 0.2 x 1 = -2.0 -2.5 ~~-5.0~~ x 0.4 x 1 = -1.0 ~~-2.0~~	+1.0V ~~0 V~~

A total of zero will not do, so make say h=0.75 ft (this is the reason for the alterations in the sheets). The + 1.0V total will appear as - 1.0V at the output of the spare amplifier used to check this derivative. The inputs quantities are

$$\begin{pmatrix}\text{source}\\ \text{voltage}\end{pmatrix} \times \begin{pmatrix}\text{potentiometer}\\ \text{setting}\end{pmatrix} \times \begin{pmatrix}\text{amplifier}\\ \text{input}\\ \text{coefficient}\end{pmatrix}$$

(9) The Paper Check

The standardised equation is

$$\frac{dh}{dt} = \frac{K}{A} H - \frac{K}{A} h - \frac{f}{A}$$

with

$$K = \frac{1}{15} \quad H= 3 \quad H = 10 \quad h = 0.75 \quad f = 0.1$$

So

$$\frac{dh}{dt} = \frac{1}{50} - \frac{0.75}{150} - \frac{1}{100} = 0.005 \text{ ft/sec}$$

The Scale of this derivative is $\frac{1}{\beta t}\left(\frac{10h}{3}\right)$, giving a voltage $\frac{0.005}{1/60} \times \frac{10}{3} = 1.00\text{V}$ which agrees with the derivative sheet.

Note:- In an integrator with a time constant of 1 sec operating in real time ($\beta = 1$) the input and output scales are the same apart from the time dimensions (indicated by t above and immaterial numerically). If $\beta < 1$ (problem takes less time i.e. it has been speeded up) then consideration of the integrating action shows that more input voltage will be required to obtain a given output voltage in the shorter time - hence the $\frac{1}{\beta}$ in the input scale factor.

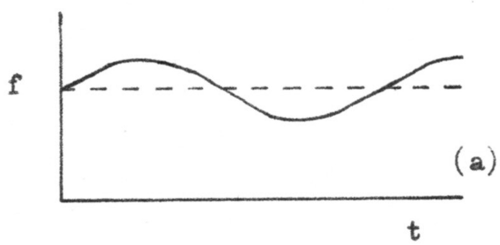
[Should the integrator time constant be T seconds (for a x 1 input) then $\frac{T}{\beta}$ would replace $\frac{1}{\beta}$].

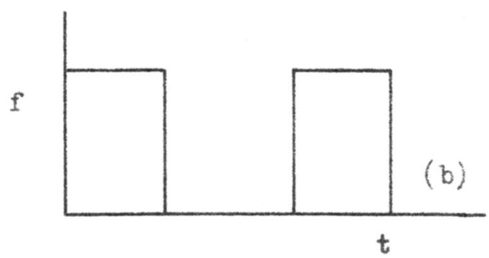
In setting an Initial Comdition, the I.C. and output scales are the same (apart from the sign) - it is really then a 1:1 inverter.

Patching on the Lan-Alog.

Switches	A$\int$	BΣ	CΣ	DΣ
Patching	Ak-Ap	Bl-Bq	Cl-Cq	Dl-Dq
	Ah-Am	Bh-Bm	Ch-Cm	Dh-Dm
	Aa-Ac	Ba -Be	Ca-Ce	Ar-Dc
	Ad-Ae	Bd-Ag	Cd-Aj	Dd-Af

The programme can be extended to cover various cases of cyclic variation of the outflow. Thus a combination of D.C. voltage and the sinusoidal output of a low frequency oscillator would represent the loading of (a), while a square wave generator would represent the heavy intermittent loadings of (b). The latter could also be achieved by manual





switching of the computer reference supply to potentiometer (2). (It might be necessary to make β nearer $\frac{1}{1}$ than $\frac{1}{60}$ and make passable accurate manual switching feasible).

Thus the maximum loading for a given tank might be found, or by altering K, H or A a tank might be designed to meet a given loading requirement.

A SECOND ORDER (2 energy store) PROBLEM

(1) The Problem

To find x for various K,F,M and P.

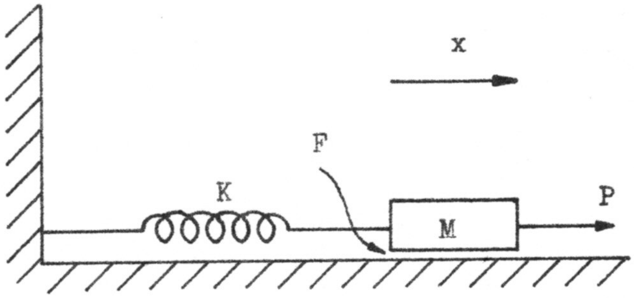
Data given:-

M = 0.05 to 0.2 slugs

K = 0 to 80 lb f/ft

F = 0 to 20 lb f/(ft/sec)

P = 0 to 100 lb f (variable waveform)



(2) The Physical Equation

$$P - F\frac{dx}{dt} - K\,x = M\frac{d^2x}{dt^2}$$

(3) The Standardised Equations

$$\ddot{x} = \frac{P}{M} - \frac{K}{M}x - \frac{F}{M}\dot{x}$$

$$\frac{d}{dt}\dot{x} = \ddot{x}$$

$$\frac{d}{dt}x = \dot{x}$$

(2nd Order Problem)

(4) Scaling

Preliminary calculations

$$\omega_{o_{max}} = \sqrt{\frac{K\ max}{M\ min}} = \sqrt{\frac{80}{0.05}} = 40 \text{ rad/s} \doteq 6\tfrac{1}{2} \text{c/s.}$$

$$F_{crit_{max}} = 2\sqrt{M\ max\ K\ max} = 2\sqrt{80 \times 0.2} = 8 \text{ lb f/(ft/sec)}$$

$$x_{max} = \pm \frac{100}{80} = \text{say} \pm 2\tfrac{1}{2} \text{ ft to allow for overshoot}^{**}$$

$$\dot{x}_{max} = \omega_{max}\ x_{max} = 100 \text{ ft/sec}$$

$$\ddot{x}_{max} = \omega^2_{max}\ x_{max} = 4000 \text{ ft/sec}^2$$

Hence scaled variables are

$$\left(\frac{\ddot{x}}{400}\right) \cdot \left(\frac{\dot{x}}{10}\right) \text{ and } \left(\frac{x}{0.25}\right)$$

(5) The Scaled Equations

$$\left(\frac{\ddot{x}}{400}\right) = \frac{P}{M}(V)^{*}\frac{1}{V \times 400} - \frac{K}{M}\left(\frac{x}{0.25}\right)\frac{1}{4 \times 400} - \frac{F}{M}\left(\frac{\dot{x}}{10}\right)\frac{10}{400}$$

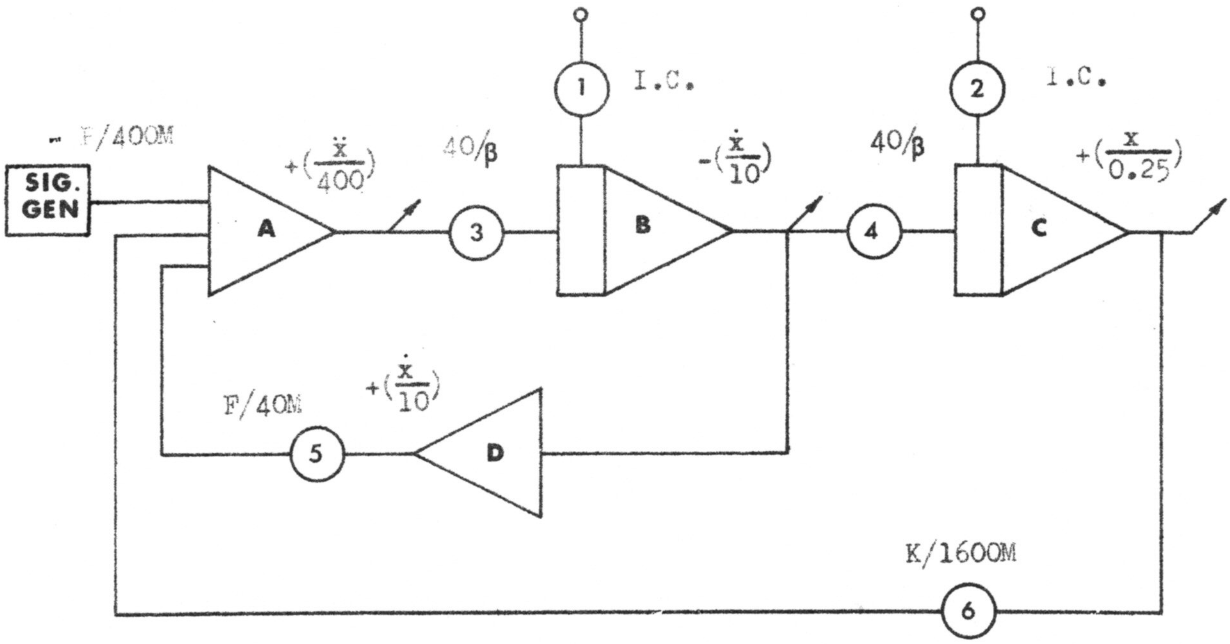
$$\frac{d}{d\tau}\left(\frac{\dot{x}}{10}\right) = \left(\frac{\ddot{x}}{400}\right)\frac{40}{\beta}$$

* V is the Signal Generator voltage.

$$\frac{d}{d\tau}\left(\frac{x}{0.25}\right) = \left(\frac{\dot{x}}{10}\right)\frac{40}{\beta}$$

** This is not the absolute x_{max}, since if K = 0 $x_{max} = \infty$. The programme will caterfor weakened spring if the applied force is also reduced; other wise a modification to the programming to allow for a larger x_{max} will be required.

(6) Computer Flow Diagram



(7) Initial Conditions

Let

P = 50 lb f

F = 1 lb f/(ft/sec) $\qquad \dot{x}$ = 30 ft/sec

K = 60 lb f/ft $\qquad$ x = 1.5 ft

M = 0.08 slug

(8) Assignment Sheets

Since no potentiometer is available to adjust P, the signal generator output must be variable (it usually is). We must know how to set it - this may be deduced from the coefficient P/400 MV , which must be exactly 1 or 10 in the absence of a pot.
Thus the required voltage (for a x 1 input) is V= P/400M = P/32 when M = 0.08 = 1.56 Volts(-) when P = 50. An external potentiometer of say 5 kΩ and the - 10V Computer voltage or a transistor power supply unit or even approximately a 1.5 volt torch battery could be used for the static check.

(2nd Order Problem)

(a) Potentiometers

Pot. No.	Parameter	Settings Static Check	Run 1	Run 2
1	I.C. of $-(\frac{\dot{x}}{10})$	+ 0.30		
2	I.C. of $+(\frac{x}{0.25})$	- 0.60		
3	$40/\beta$	0.4 x 10		
4	$40/\beta$	0.4 x 10		
5	F/ 40M	0.312		
6	K/ 1600M	0.468		

β = 10 has been used.

(b) Amplifiers and (c) Derivatives

Calculations from the scaled Equations:-

$$(\frac{\ddot{x}}{400}) = \frac{50}{0.08}\,\frac{1}{400} - \frac{60}{0.08}\,(6)\frac{1}{1600} - \frac{1}{0.08}\,(3)\,\frac{1}{40}$$

$$= 1.56 - 2.82 - 0.94$$

$$= -2.20 \text{ volts.}$$

$$\frac{d}{d\tau}(\frac{\dot{x}}{10}) = (\frac{\ddot{x}}{400})\frac{40}{\beta} = -2.20 \times 4 = -8.80\text{volts}$$

$$\frac{d}{d\tau}(\frac{x}{0.25}) = (\frac{\dot{x}}{10})\frac{40}{\beta} = 3.0 \times 4 = 12.0 \text{ volts}^*$$

Notice that the second equation cannot be solved directly before the first.

*It is possible for such a large signal to appear at the input to an integrator - it merely means a rapid rate of change of the integrator output: it is best checked by reducing pot.4 temporarily to 0.2 or putting the signal into a x 1 input on the checking amplifier, thus seeing 6.0 or 1.2 volts as the check.

Hence (b) the amplifier sheet:-

Amplifier No.	Output	Static Check	Run 1	Run 2
A	$+(\frac{\ddot{x}}{400})$	-2.20		
B	$-(\frac{\dot{x}}{10})$	-3.00		
C	$+(\frac{x}{0.25})$	+6.00		
D	$+(\frac{\dot{x}}{10})$	+3.00		

and (c) the Derivatives

Amplifier No.	Inputs	Static Check
B	$+ \frac{d}{d\tau}$ of $(\frac{\dot{x}}{10})$	- 8.80
C	$- \frac{d}{d\tau}$ of $(\frac{x}{0.25})$	- 12.0

(9) The Paper Check

Calculations from the Standardised Equations:-

$$\ddot{x} = \frac{P}{M} - \frac{K}{M}x - \frac{F}{M}\dot{x}$$

$$= \frac{50}{0.08} - \frac{60}{0.08} \times 1.5 - \frac{1}{0.08} \times 30$$

$$= 625 - 1125 - 375$$

$$= -875 \text{ ft/sec}^2$$

$$\frac{d}{dt}\dot{x} = \ddot{x} = -875 \text{ ft/sec}^2$$

$$\frac{d}{dt}x = \dot{x} = 30 \text{ ft/sec}$$

Whence we have

Amplifier	Input/Output	Value	Scale	Voltage	Check with 8(b) &(
A	$\ddot{x}$	- 875	$+ (\frac{\ddot{x}}{400})$	+2.19	*
B	$\frac{d}{dt}\dot{x}$	- 875	$-\frac{1}{\beta t}$ of $- (\frac{\dot{x}}{10})$	-8.75	*
C	$\frac{d}{dt}x$	+ 30	$-\frac{1}{\beta t}$ of $+ (\frac{x}{0.25})$	-12.0	✓

*Slight discrepancy due to 3 figure working.

This programme can be used:-

(i) to show the variation of natural frequency with M or K (with little or no damping);

(ii) to show the phase relationships of position velocity and acceleration in Simple Harmonic motion (no damping: look at amplifiers A, D and C to get the sign right);

(iii) to show for a given M and K the reduction in natural frequency as F is increased until finally oscillation ceases when the system is critically damped;

(iv) to obtain the response to a step input i.e. a suddenly applied force (a square wave generator or a manually operated switch in the P line will do this), showing , for instance, no overshoot (or reversal of velocity) when critically or over damped, but oscillations when underdamped;

(v) to show, with sinusoidal variations of P (from an L.F. oscillator), a linear build up of oscillations - without limit, except for the 10V maximum of the computer - when P 's frequency resonates with the system (no damping)

(vi) to show a limited build up, under similar conditions, if friction is present and

(vii) to show a beat effect if, with little or no friction, the frequency of P is just above or below resonance.

THE SECOND ORDER PROBLEM.

Switches:

A $-\Sigma$ B $-\int$ C $-\int$ D $-\Sigma$

Aa - Ac	Bb - Bc	Sig. Gen. to Ae
Ad - Bj	Bd - Cj	(caution : 10V max)
Ar - Cc	Br - De	Cr - Ds
Cd - Bg	Dr - As	Du - Dv
Br - Dc	Au - Av	Dt - 1MΩ - Ah
Dd - Cg	At - Af	

CHAPTER 5.

SLAVING — TWO LAN-ALOG COMPUTERS

5.1. Introduction

In previous chapters the examples given have all been soluble on one Lan-Alog computer. It is the purpose of this chapter to give the solution of a more complex problem in order to illustrate the full capability of the machine.

Two Lan-Alog units are operated as one computer, giving an overall complement of eight amplifiers, all of which may be used as integrators, or summer/inverters as desired.

When looking through the example which follows the reader will appreciate that the logical and systematic method of programming described in chapter 3 is essential.

5.2. Master and Slave

To operate as Master and Slave the two Lan-Alogs must be joined by the special synchronizing cable which is plugged into the seven-way socket in column D. The machines require individual power inputs, usually from the same power supply. One machine is switched to MASTER (this machine will be called LA-4 (1)) and the other is switched to SLAVE (this machine will be known as LA-4 (2)). The effects of the mode switches and the Auto/Manual toggle switch are then as follows:-

LA-4 (1) (Master)

Master-Balance-Slave switch —— to MASTER.

NOTE: Switching to BALANCE enables amplifiers of both LA-4 (1) and LA-4 (2) to be balanced.

Reset-Compute-Hold switch —— Operates both LA-4 (1) and LA-4 (2).

Auto/Manual toggle switch, Slow/Fast switch and Speed Control Potentiometer all operate both LA-4 (1) and LA-4 (2).

LA-4 (2) (Slave)

Master-Balance-Slave switch —— To SLAVE.

NOTE: This switch remains in this position throughout amplifier balancing, checking-out and problem runs.

Reset-Compute-Hold, Auto/Manual and Slow/Fast switches and Speed Control Potentiometer are not active.

The MASTER has complete control over the SLAVE.

5.3. Car Suspension Problem

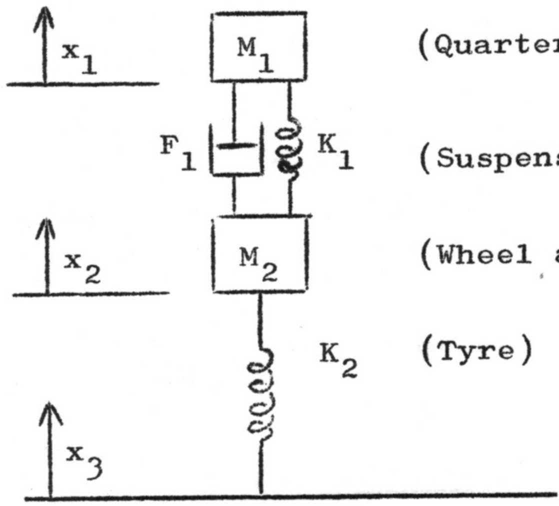
5.3.1. Statement of Problem

a) Description

One wheel of the car is to be considered, which means that we have in effect cut the car into four quarters. Of course the problem is simplified by doing this because we have immediately established a two dimensional motion. The problem would have been extremely complex if we had considered pitching and rolling of the vehicle and we would have required a much larger computer to establish the solution.

A second simplification which we will impose is to neglect the damping effect of the tyre. The tyre will thus be considered as possessing the property of a simple spring.

Diagramatic representation of the Physical System:



Note: In the static position under the action of gravity x_1 and x_2 are both zero. x_3 is measured from ground level.

b) Problem Constants

M_1 = 600 lbm

M_2 = 40 lbm (wheel) + 24 lbm (half axle) = 64 lbm

K_1 = 1000 lbf/ft

K_2 = 6000 lbf/ft

F_1 = 50 lbf/ft/s

c) Required Outputs

It is desired to investigate the motion of the car body (remembering that we are thinking only of two dimensional motion) when the car mounts a step. Further to investigate the motion if the damping constant of the suspension is increased to 100 lbf/ft/s. Only x_1, x_2, $\dot{x}_1$ and $\dot{x}_2$ will be required. Accelerations will not be required explicitly.

d) Problem Variables

Reasonable limits will have to be imposed on the problem variables for scaling purposes. The following will be used:

Magnitude of x_1 to be $\leqslant$ 1 ft.
Magnitude of x_2 to be $\leqslant$ 1 ft.
Magnitude of velocity $\dot{x}_1$ to be $\leqslant$ 10 ft/s.
Magnitude of velocity $\dot{x}_2$ to be $\leqslant$ 50 ft/s } See Section 5.3.4.
Magnitude of step $x_3 \leqslant$ 2"

5.3.2. Physical Equations

$$M_1 . \frac{d^2 x_1}{dt^2} = K_1 . (x_2 - x_1) + F_1 . (\dot{x}_2 - \dot{x}_1)$$

$$M_2 . \frac{d^2 x_2}{dt^2} = K_1 . (x_1 - x_2) + F_1 . (\dot{x}_1 - \dot{x}_2) - K_2 . (x_2 - x_3)$$

5.3.3. Standardised Equations

$$\frac{d}{dt}(\dot{x}_1) = \frac{K_1}{M_1}.(x_2) - \frac{K_1}{M_1}.(x_1) + \frac{F_1}{M_1}.(\dot{x}_2) - \frac{F_1}{M_1}.(\dot{x}_1)$$

$$\frac{d}{dt}(x_1) = (\dot{x}_1)$$

$$\frac{d}{dt}(\dot{x}_2) = \frac{K_1}{M_2}.(x_1) - \frac{K_1}{M_2}.(x_2) + \frac{F_1}{M_2}.(\dot{x}_1) - \frac{F_1}{M_2}.(\dot{x}_2) - \frac{K_2}{M_2}.(x_2) + \frac{K_2}{M_2}.(x_3)$$

$$\frac{d}{dt}(x_2) = (\dot{x}_2)$$

5.3.4. Scaling

Various reasonable approximations may be made to establish rough figures for the maximum values of the problem variables. With a fairly complex problem it will be appreciated that it is difficult to work out maximum values for the problem variables in a precise fashion. It is best to calculate the value which one thinks will not be exceeded and then double this value (or increase by 50% or treble as one thinks fit) for the purposes of scaling. The amount by which one over-estimates is dependent upon the degree of guess-work present in the initial calculation. It is always better to over-estimate rather than under-estimate maximum values so that at least some sort of measurable output may be obtained. Any readable output (however small), is better than producing an overload, since re-scaling can be carried out with confidence as soon as one has some idea of the maximum voltage output which is present.

It is worth noting that it is advisable to use consistent units (namely lbf., slugs, ft/s. etc.) in making all preliminary calculations. So that $M_1 = \frac{600}{32.2} \simeq 18.7$ slugs and

$$M_2 = \frac{64}{32.2} \simeq 2 \text{ slugs.}$$

Maximum values for $\dot{x}_1$ and $\dot{x}_2$ have been established using approximate methods. These are shown in Section 5.3.1. (d).

Hence, our scaled variables (using maxima quoted in 5.3.1. (d)) are:-

$$\left(\frac{x_1}{1}.10\right) \qquad \left(\frac{x_2}{1}.10\right) \qquad \left(\frac{x_3}{\frac{1}{6}}.10\right) \qquad \left(\frac{\dot{x}_1}{10}.10\right) \qquad \left(\frac{\dot{x}_2}{50}.10\right)$$

Namely, $(10x_1)$ $(10x_2)$ $(60x_3)$ $(\dot{x}_1)$ $\left(\frac{\dot{x}_2}{5}\right)$

5.3.5. The Scaled Equations

$$\frac{d(\dot{x}_1)}{d\tau} = \frac{K_1}{M_1}.(10x_2).\frac{1}{10}.\frac{1}{\beta} - \frac{K_1}{M_1}.(10x_1).\frac{1}{10}.\frac{1}{\beta} + \frac{F_1}{M_1}.\left(\frac{\dot{x}_2}{5}\right).5.\frac{1}{\beta} - \frac{F_1}{M_1}.(\dot{x}_1).\frac{1}{\beta}$$

$$\frac{d(10x_1)}{d\tau} = (\dot{x}_1).10.\frac{1}{\beta}$$

$$\frac{d}{d\tau}\left(\frac{\dot{x}_2}{5}\right) = \frac{K_1}{M_2}.(10x_1).\frac{1}{10}.\frac{1}{5}.\frac{1}{\beta} - \frac{K_1}{M_2}.(10x_2)*\frac{1}{10}.\frac{1}{5}.\frac{1}{\beta} + \frac{F_1}{M_2}.(\dot{x}_1).\frac{1}{5}.\frac{1}{\beta}$$

(cont.)

$$\frac{-F_1}{M_2}\cdot\left(\frac{\dot{x}_2}{5}\right)\cdot\frac{1}{\beta} - \frac{K_2}{M_2}\cdot(10x_2)^{*}\cdot\frac{1}{10}\cdot\frac{1}{5}\cdot\frac{1}{\beta} + \frac{K_2}{M_2}\cdot(60x_3)\cdot\frac{1}{60}\cdot\frac{1}{5}\cdot\frac{1}{\beta}$$

$$\frac{d}{d\tau}(10x_2) = \left(\frac{\dot{x}_2}{5}\right)\cdot 5\cdot 10\cdot\frac{1}{\beta}$$

* One potentiometer having coefficient $\frac{(K_1+K_2)}{50M_2\beta}$ is required.

5.3.6. The Computer Flow Diagram
(This is given on page 60)

At first sight it appears that we require a larger number of potentiometers than we have available. We note that β appears in the coefficients of each of potentiometers 7,8 and 9. Potentiometer 8 has the most complex coefficient, this being $\frac{K_1}{50M_2\beta}$.

Now, $\frac{K_1}{50M_2\beta} = \frac{1000}{50.2.\beta} = \frac{10}{\beta}$ which is the same as potentiometer 7.

Clearly then a β value of 10 will eliminate potentiometers 7 and 8 and will make a coefficient of 5 necessary for potentiometer 9. We can use a 5 times input to amplifier C of LA-4 (2) to make potentiometer 9 redundant. A 0.1MΩ resistor in series with the 0.1 MΩ input resistor on amplifier LA-4 (2) C gives the 5 times input as required.
Note: It must be ascertained that no unreasonable coefficients will be necessary for any other potentiometer before using the value 10 for β. As this has been checked we can now proceed with the solution.

5.3.7. Initial Conditions

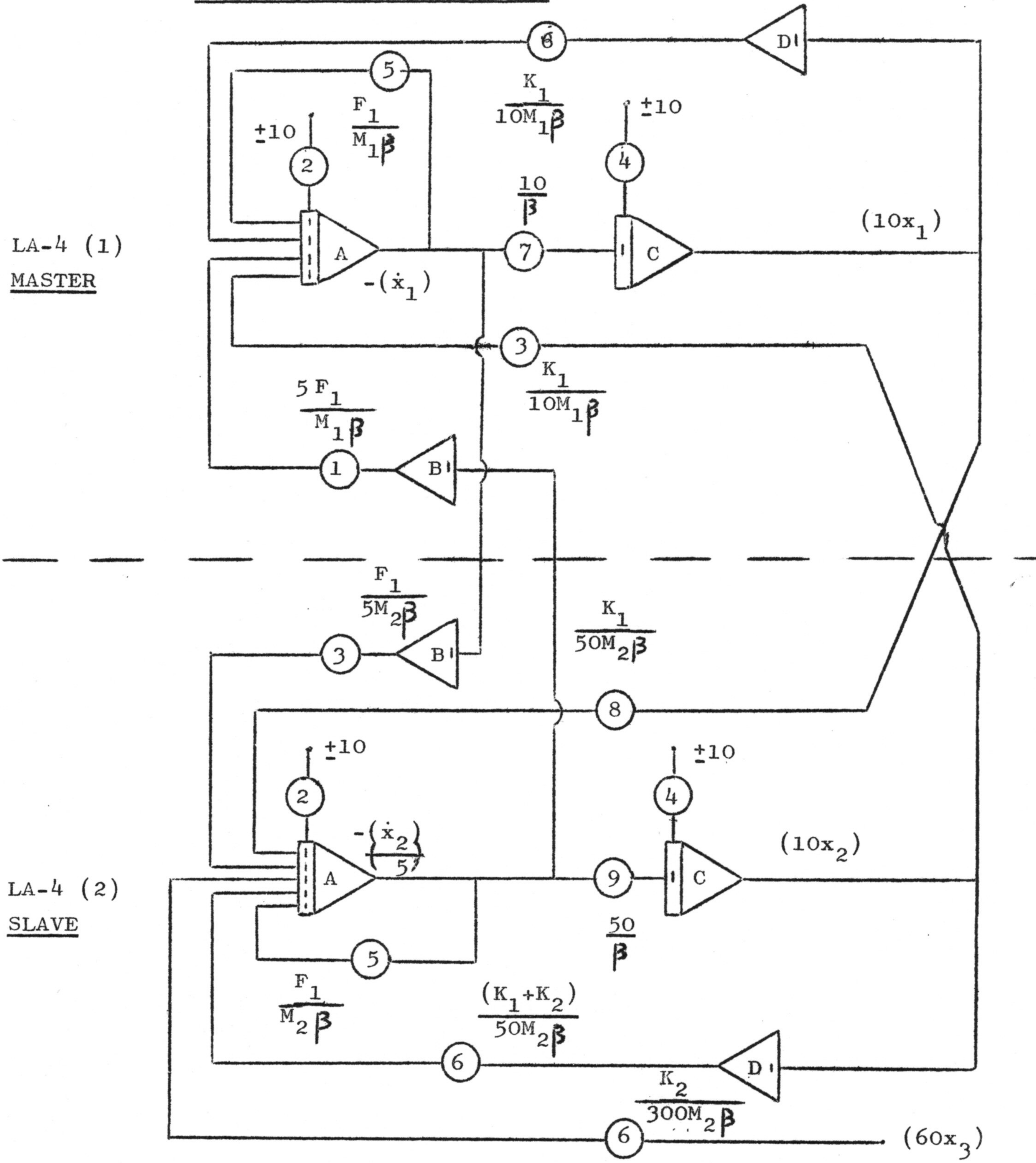
Let: $x_1 = 3''$ $\dot{x}_1 = 6$ ft/s.
$x_2 = 2''$ $\dot{x}_2 = 20$ ft/s.

5.3.8. The Assignment Sheets

We will take $\dot{F}_1$ to be 50 lbf/ft/s. It is appreciated taht if it had the value 100 lbf/ft/s, as required in part of the problem solution, the motion would be more "sluggish".

A value of 1" for x_3 will be used.

The Computer Flow Diagram



Note: Potentiometers 7,8 and 9 are not essential.

a) <u>Potentiometers</u>

Group	Ref.No.	Parameter & Value	Coefficients: Static Check (with β=10)	Coefficients: Run 1	Coefficients: Run 2
LA-4 (1)	1	$5F_1/M_1\beta = 13.36/\beta$	0.1336 x 10		0.2672x10
	2	To give I.C. of $-(\dot{x}_1)$ on Amplifier A	(+0.6)†		
	3	$K_1/10M_1\beta = 5.35/\beta$	0.535		
	4	To give I.C.of $(10x_1)$ on Amplifier C	-0.25		
	5	$F_1/M_1\beta = 2.68/\beta$	0.268		0.536
	6	$K_1/10M_1\beta = 5.35/\beta$	0.535		
LA-4 (2)	1	$(\frac{1}{2})^{‡} \times \frac{K_2}{300M_2\beta} = (\frac{1}{2}) \times \frac{10}{\beta}$	0.5	1.0	0.5
	2	To give I.C.of $-\left(\frac{\dot{x}_2}{5}\right)$ on Amplifier A	+0.4		
	3	$F_1/5M_2\beta = 5/\beta$	0.5		1.0
	4	To give I.C.of $(10x_2)$ on Amplifier C	-0.167		
	5	$F_1/M_2\beta = 25/\beta$	0.25x10		0.5x10
	6	$\frac{(K_1+K_2)}{50M_2\beta} = 70/\beta$	0.7x10		
	7	$10/\beta$	1		
	8	$K_1/50M_2\beta = 10/\beta$	1		
	9	$50/\beta$	5 ‡		

† Interpretation: Potentiometer 2 is connected to + 10 reference voltage and has a coefficient of 0.6.

‡ Due to x_3 being 1" for the Static check. (see Flow Diagram of 5.3.6.)

‡‡ Two 100 KΩ in series for input to point m of Amplifier C.

<u>Note</u>: Values are given in Run 1 and Run 2 columns only when they differ from those of the Static Check column.

b) <u>Amplifers</u>

	Amplifier	Output	Static Check Voltage	Run 1	Run 2
LA-4 (1)	A	$-(\dot{x}_1)$	-6.00		
LA-4 (1)	B	$+\left(\frac{\dot{x}_2}{5}\right)$	+4.00		
LA-4 (1)	C	$+(10x_1)$	+2.50		
LA-4 (1)	D	$-(10x_1)$	-2.50		
LA-4 (2)	A	$-\left(\frac{\dot{x}_2}{5}\right)$	-4.00		
LA-4 (2)	B	$+(\dot{x}_1)$	+6.00		
LA-4 (2)	C	$+(10x_2)$	+1.67		
LA-4 (2)	D	$-(10x_2)$	-1.67		

c) <u>Derivatives</u> (from the <u>scaled</u> equations)

	Amplifer	Inputs	Total
LA-4 (1)	A	-2.5x0.535x1 + 1.67x0.535x1 +4x0.1336x10 - 6x0.268x1	+3.3
LA-4 (1)	C	-6x1	-6

	Amplifier	Inputs	Total
LA-4(2)	A	2.5x1 - 1.67x0.7x10 + 6x0.5x1 -4x0.25x10 + 10x0.5x1	-11.2*
	C	-4x0.5x10	-20**

* Check by using an extra 1MΩ in series with a 1MΩ input resistor to give a 0.5x input. Thus -5.6 is voltage to check.

** Check on a 1x input rather than on the correct input of 10x. Thus - 2 volts is the value to be checked.

These will appear as -3.3, +6, +5.6 and +2 volts respectively at the output of the amplifier used to check them. In practice, amplifier D of LA-4 (2) would be used to check amplifiers A and C of LA-4 (1). Amplifier D of LA-4 (2) would then be returned to its original condition. Then, amplifier D of LA-4 (1) would be used to check amplifiers A and C of LA-4 (2). Similarly, this amplifier D would then be returned to its original condition.

5.3.9. The Paper Check

From the standardised equations we have:-

$$\frac{d}{dt}(\dot{x}_1) = \frac{1000}{18.7}\cdot\left(\frac{1}{6}\right) - \frac{1000}{18.7}\left(\frac{1}{4}\right) + \frac{50}{18.7}\cdot(20) - \frac{50}{18.7}\cdot(6) = 33 \text{ ft/s}^2$$

$$\frac{-d}{dt}(x_1) = -6 \text{ ft/s.}$$

$$\frac{d}{dt}(\dot{x}_2) = \frac{1000}{2}\left(\frac{1}{4}\right) - \frac{1000}{2}\left(\frac{1}{6}\right) + \frac{50}{2}\cdot(6) - \frac{50}{2}\cdot(20) - \frac{6000}{2}\cdot\left(\frac{1}{6}\right) + \frac{6000}{2}\left(\frac{1}{12}\right) = -560 \text{ ft/s}^2$$

$$\frac{-d}{dt}(x_2) = -20 \text{ ft/s.}$$

Hence we have:-

	Amplifier	Derivative	Value	Scale (β=10)	Voltage***
LA-4 (1)	A	$\frac{d}{dt}(\dot{x}_1)$	+33	$\frac{1}{\beta}$ of $(\dot{x}_1)$	+3.3
	C	$\frac{-d}{dt}(x_1)$	-6.0	$\frac{1}{\beta}$ of$(10x_1)$	-6.0
LA-4 (2)	A	$\frac{d}{dt}(\dot{x}_2)$	-560	$\frac{1}{\beta}$ of $\left(\frac{\dot{x}_2}{5}\right)$	-11.2
	C	$\frac{-d}{dt}(x_2)$	-20	$\frac{1}{\beta}$of$(10x_2)$	-20.0

*** At input to integrator : Output from checking amplifier will be of opposite polarity.

These voltages are now checked against those from 5.3.8. (c)

The problem is now ready for the computer using the procedure given in Chapter 3 (Sections 3.2.2. and 3.3.).

5.3.10. The Patching Diagram

Switches :- LA-4 (1) and LA-4 (2)
A and C to $\int$
B and D to Σ

Note: Since 0.1μF feedback capacitors are used on all integrators the problem is solved in real time (i.e.) effect of value of 10 for β is cancelled due to these capacitors.

LA-4 (1)

Ac - Br	Au - Av	Cr - Meter (y)
Ad - Ag	Ar - Ce	Cr* - 'Scope y deflection
Ae - Dt	Ba - Bc	Cv - 'Scope earth
Af - At	Bh - Bm	Cr - De
Ag - Ad	Bl - Bq	Db - Dc
Ah - Am	Cd - 1MΩ - Ah	Dh - Dm
Aj - Bd	Ch - Cm	Dl - Dq
Ak - Ap	Cj - Dd	Dr - Ds
As - Ar	Ck - Cp	Du - Dv

LA-4(1) to LA-4(2)	IAr - IIBe IBe - IIAr ICc - IICr ICr - IIAe		[I refers to LA-4 (1) II refers to LA-4 (2)]

LA-4 (2)	Aa - Ac	Ar - 100kΩ+ - Cg	Cj - Dd
	Ad - 1MΩ+ - Ah	Au - Av	Ck - Cp
	Af - Cd	At - 100kΩ+ - Ah	Cr* - De
	Ag - Dt	Ba - Bc	Db - Dc
	Ah - Am	Bh - Bm	Dh - Dm
	Aj - Bd	Bl - Bq	Dl - Dq
	Ak - Ap	Br - Cc	Dr - Dt
	Ar - As	Ch - Cm	Du - Dv

* x_1 (LA-4 (1) amplifier C output) and x_2 (LA-4 (2) amplifier C output) can be viewed simultaneously on a 'scope with multiple inputs.

\+ These are external resistors. In all, two of 1MΩ and two of 100kΩ are required for this problem.

This program is suitable for repetitive operation at about 0.1 Hz.

5.3.11. Uses of the Program

Many variations of initial conditions, (i.e.) values of x_1, $\dot{x}_1$, x_2, and $\dot{x}_2$ at the instant of mounting the step, may be tried. Maxima for these variables are shown in 5.3.1. (c).

If the problem is run using the static check potentiometer settings of 5.3.8. (a) the following initial conditions hold:-

$x_1 = 3"$ $\qquad \dot{x}_1 = 6$ ft/s.

$x_2 = 2"$ $\qquad \dot{x}_2 = 20$ ft/s.

Also for this run, $F_1 = 50$ lbf/ft/s and step $x_3 = 1"$

Potentiometer coefficients for two other runs are shown in 5.3.8. (a) These are:

<u>Run 1</u> Same initial conditions as above, but step x_3 altered to 2" with F_1 remaining at 50 lbf/ft/s.

<u>Run 2</u> Again, same initial conditions, but step x_3 returned to 1" with F_1 taking a value of 100 lbf/ft/s.

A further possibility for this program is application to the case where the road is "corrugated" say ± 2" with crests 10 ft. apart (sine generator input for x_3). It would be possible, with suitable rescaling of the variables, to modify the Standardised Equations to solve this problem. The speed at which the car travels along the road will of course have an appreciable affect on the solution.

CHAPTER 6

THE MULTIPLIER UNIT.

6.1. Introduction

The Lan-Alog Multiplier unit (LA-4M) is an additional panel which extends the capabilities of the basic Lan-Alog 4 computer to non-linear examples and problems. The Multiplier unit includes the following components:-

One electronic multiplier module.

Three operational amplifiers, with resistive feedback.

Five coefficient potentiometers, two "earth-free".

The multiplier module is a compact, solid-state, encapsulated unit giving full four-quadrant operation. The circuit gives an accuracy of ± 0.3% of the output range, which is ± 10V, at 20°C. The multiplier can be used in ambient temperatures from - 20°C to + 65°C.

The three amplifiers are of the standard SA-1 type, as used in the Lan-Alog computer, giving an open-loop gain of 50,000 times and an input impedance of about 100 kΩ with a thermal drift of less than 10 μV/°C.

The layout of the multiplier panel is quite similar to that of the basic computer. The three amplifier symbols are labelled E,F and G and the rectangular multiplier symbol is marked M. Wherever possible the socket letters correspond on the multiplier and computer units. Each amplifier has a summing junction with two resistors, both of 1MΩ. In addition amplifiers E and F have an isolated 100 kΩ resistor while amplifier G has an isolated 20 kΩ resistor. The use of these resistors in the feedback or input circuits can provide a wide variety of scale factors.

6.2. Controls

There are three main controls on the multiplier panel, seven balance potentiometers and five coefficient potentiometers (numbered from 7 through 11).

The multiplier unit has a 5-way socket for the power lead and must be supplied from one output of the LP 402/LA power unit, under control of the ON/OFF switch.

A four-way switch is marked COMPUTE/SCALE/ZERO 1 /ZERO 2. It is desirable to Balance each amplifier before any period of use. For this purpose switch to ZERO 1, ZERO 2 or SCALE, join the output of each amplifier to the meter, in turn, and adjust the appropriate balance potentiometer to give a zero voltage reading.

Three adjustments to the multiplier circuit should also be checked, with the following switch positions:-

(i) ZERO 1, - ZERO 1 potentiometer must be adjusted for zero output voltage from socket Mz.

(ii) ZERO 2, - There are two potentiometers for this purpose. The potentiometer with screw-driver adjustment is for coarse zero control. ZERO 2 potentiometers must be adjusted to give zero output voltage from socket Mz.

(iii) SCALE - The Scale Factor control must be adjusted to give an output of exactly 10 V from socket Mz.

After these adjustments have been made the four-way switch must be put to the COMPUTE position.

The Multiply/Divide function switch has two positions marked X and ÷ , and its use is described below.

6.3. Multiplication

If the inputs to the multiplier are voltages X and Y and the function switch is put to "X", then the output is a voltage, $Z = \frac{X.Y}{10}$,

where inputs X and Y may both lie within the full dynamic range of $\pm$ 10V.

The operation of a quarter-squares electronic multiplier may be shown by the following equations:-

$$10Z = X.Y = \frac{1}{4}\left((X+Y)^2 - (X-Y)^2 \right)$$

since $(X+Y)^2 = Y^2 + X^2 + 2XY$

and $(X-Y)^2 = Y^2 + X^2 - 2XY$

This result may be achieved by three operational amplifiers and two square law devices. In the Lan-Alog 4M the multiplier module employs a miniature light source and a photo-sensitive resistor as the square law device.

6.4. Division

When the function switch is put to ÷ the multiplier is effectively placed in the feedback circuit of amplifier G and it is then possible to obtain electronic division. Under this condition only two amplifiers (E and F) are available for other functions.

The numerator voltage must be applied to socket Mx and the denominator voltage to My. The resultant division is then obtainable at the output Mz where now, $Z = -\frac{10X}{Y}$.

It should be noted that for division input X may vary over the full dynamic range of ± 10V, but input Y may be only between 0 and + 10V. This may require that the denominator voltage is fed through an inverting amplifier before the multiplier.

6.5. Other Applications

The multiplier M may be used as a squaring circuit by feeding the same voltage signal into both inputs Mx and My.

It is worth noting that the Lan-Alog multiplier panel can be used apart from the basic Lan-Alog computer (but with the correct LP 402 twin power unit) for the purposes of electronic summing, multiplication and division. The LA-4M may be economical for these purposes in some industrial and research applications.

6.6. Host-Parasite Example

The situation under investigation is as follows:-

In a parasitic relationship in which the birth of a parasite (namely an increase in the parasite population) causes a decrease in the host population, the rate of host population growth or decay depends on the number of living hosts and the number of encounters between host and parasite.

In analysing the system the following assumptions will be made:

a) In the absence of parasites, the net rate of increase in host population is exponential, the food supply being unlimited.

b) In the absence of hosts, the rate of parasite decrease is also exponential.

c) The rate of encounter between hosts and parasites is constant.

6.6.1. Physical Equations

$$\frac{dH}{dt} = nH - kPH$$

$$\frac{dP}{dt} = -qP + kPH$$

where H = host population (varies with time)
P = parasite population (varies with time)
n = natural increase rate of hosts
q = rate of natural parasite death
k = rate of parasite egg-laying leading to host death
t = time

6.6.2. Scaling

Suppose that, initially, there are 20 hosts and 40 parasites and that the pattern of the relationship between them will be established by the time the host population reaches 1000 and/or the parasite population reaches 500.

The hosts increase at a rate of 2.5% per hour (in the absence of parasites) and the decay rate for parasites is 5% per hour (in the absence of hosts). Further, it will be assumed that there is one encounter between host and parasite per 1600 hours.

Hence,

$$n = 0.025/\text{hour}$$
$$q = 0.05/\text{hour}$$
$$k = 6.25 \times 10^{-4} \Big\} \frac{\text{host hour}}{\text{parasite hour}}$$

6.6.3. Scaled Variables

$$\left(\frac{H}{1000} \cdot 10\right) = \left(\frac{H}{100}\right)$$

$$\left(\frac{P}{500} \cdot 10\right) = \left(\frac{P}{50}\right)$$

$$\left(\frac{P \cdot H}{1000 . 500} \cdot 10\right) = \left(\frac{P.H}{5x10^4}\right)$$

6.6.4. Scaled Equations

From the Standardised Equations (these are identical to the Physical Equations in this particular problem) :-

$$\frac{d}{d\tau}\left(\frac{H}{100}\right) = \frac{n}{\beta} \cdot \left(\frac{H}{100}\right) - \frac{k}{\beta} \cdot 5.10^2 . \left(\frac{PH}{5x10^4}\right)$$

$$\frac{d}{d\tau}\left(\frac{P}{50}\right) = \frac{-q}{\beta} \cdot \left(\frac{P}{50}\right) + \frac{k}{\beta} \cdot 10^3 . \left(\frac{P \cdot H}{5 \times 10^4}\right)$$

6.6.5. Flow Diagram

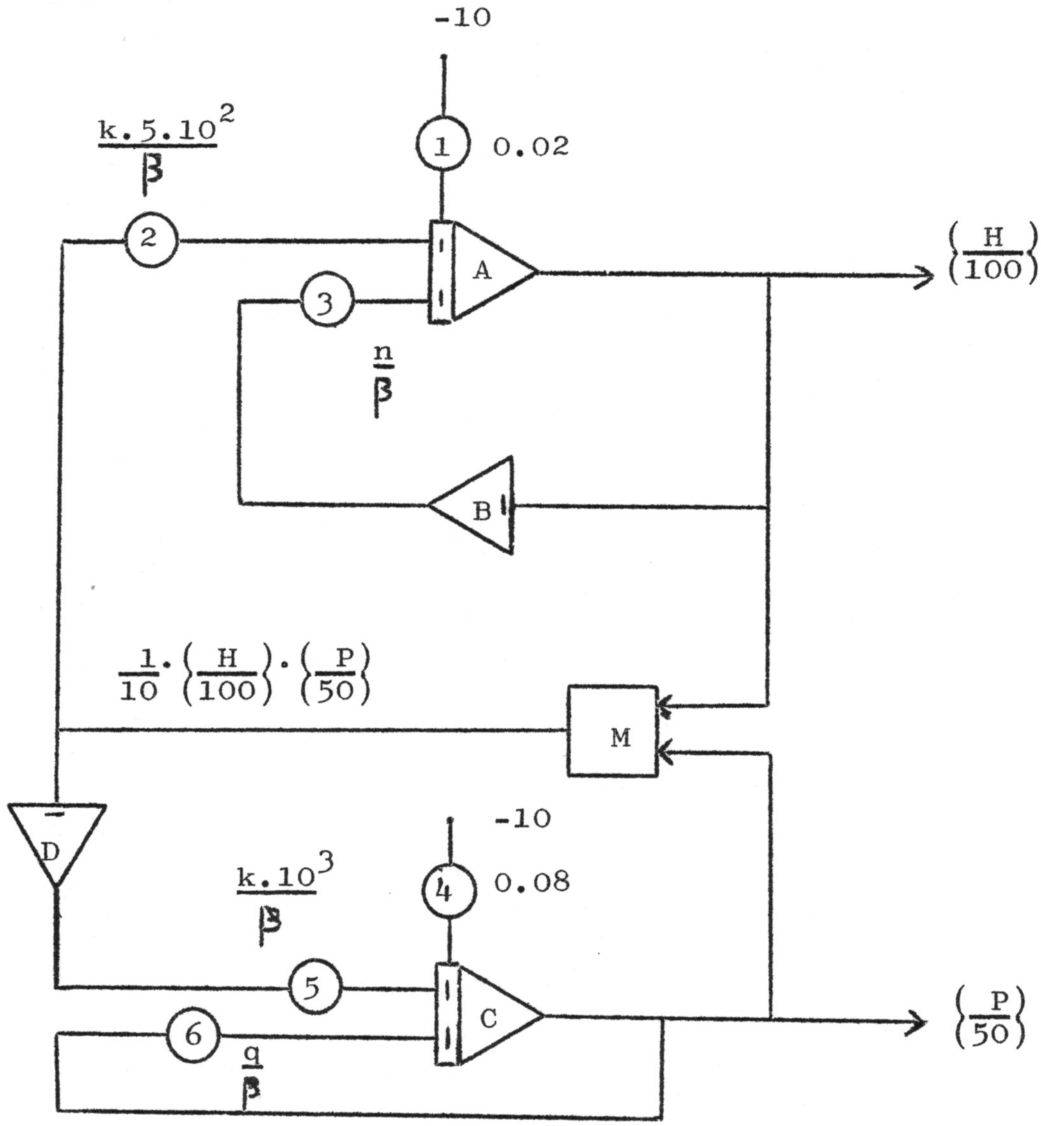


Fig. 6.1.

Assignment and Check Sheets are omitted from the problem, but can be drawn up by the reader.

6.6.6. Potentiometer Settings

A time scaling factor (β) of $\frac{1}{10}$ is used, which speeds up the solution of the problem by 10 times. It must be remembered that in addition to this speeding up of the solution, there is also an implied time scaling which relates computer time (in seconds) to problem time (in hours). Thus, 1 second of computer time is equivalent to 10 hours of real time.

6.6.7. Table of potentiometer settings

Ref. No.	Parameter and Value	Coefficient (β = 0.1)
1		0.02
2	$\frac{500k}{\beta} = \frac{0.313}{\beta}$	3.13
3	$\frac{n}{\beta} = \frac{0.025}{\beta}$	0.25
4		0.08
5	$\frac{1000k}{\beta} = \frac{0.625}{\beta}$	6.25
6	$\frac{q}{\beta} = \frac{0.05}{\beta}$	0.5

6.6.8. The Patching Diagram

Switches: Multiplier Unit to Compute and X.
Amplifiers A and C to $\int$.
Amplifiers B and D to Σ.

Ab - Ac
Ad - Aj
Ak - An
Ah - Am
Au - Av
Ag - Bd
Ae - Cd

As - Dr

At - Cg
Ar - Be

Ar - Mx
Bc - Mz
Bh - Bm
Bl - Bq
Br - Cc
Ch - Cm
Ck - Cn

Cf - Dt

Cr - Ds
Cb - Dc

Cj - Dd
Cr - My
De - Mz
Dh - Dm
Dl - Dq
Du - Dv
Ar - Display
(Number of hosts)
Cr - Display
(Number of parasites)
Em - Eq
Fm - Fq
Gm - Gq

6.7. Solution of Mathieus' Equation

The behaviour of many physical systems, including mechanical systems with sinusoidal excitation, is described by this equation.

One way of expressing this equation is as follows:

$$\frac{d^2x}{dt^2} + (k - 2n.\cos wt).\ x = 0$$

The particular case where $k = 2n$, $w = 2$ and $0 \leqslant k \leqslant 5$ enables us to study stability and instability.

We shall therefore obtain solution of the equation

$$\frac{d^2x}{dt^2} + k.(1 - \cos 2t).\ x = 0,$$ and insert arbitrary

initial conditions of $x = 1$, $\dot{x} = 0$ at $t = o$.

It is obviously necessary to generate a function $y = 1 - \cos 2t$ and so we will firstly establish a flow diagram to meet this requirement.

$y = 1 - \cos 2t$ ....(i)

$\dot{y} = 2 \sin 2t$ ....(ii)

$\ddot{y} = 4 \cos 2t$, which becomes (by using equation (i))

$\ddot{y} = 4.(1 - y)$..........(iii)

6.7.1. Scaling

From (i) Maximum value of $y = 2$

From (ii) Maximum value of $\dot{y} = 2$

$\therefore$ Scaled variables are, $(\frac{y}{2} \cdot 10) = (5y)$

and, $(\frac{\dot{y}}{2} \cdot 10) = (5\dot{y})$

6.7.2. Standardised Equations

$$\frac{d(\dot{y})}{dt} = 4.\Big(1 - (y)\Big)$$

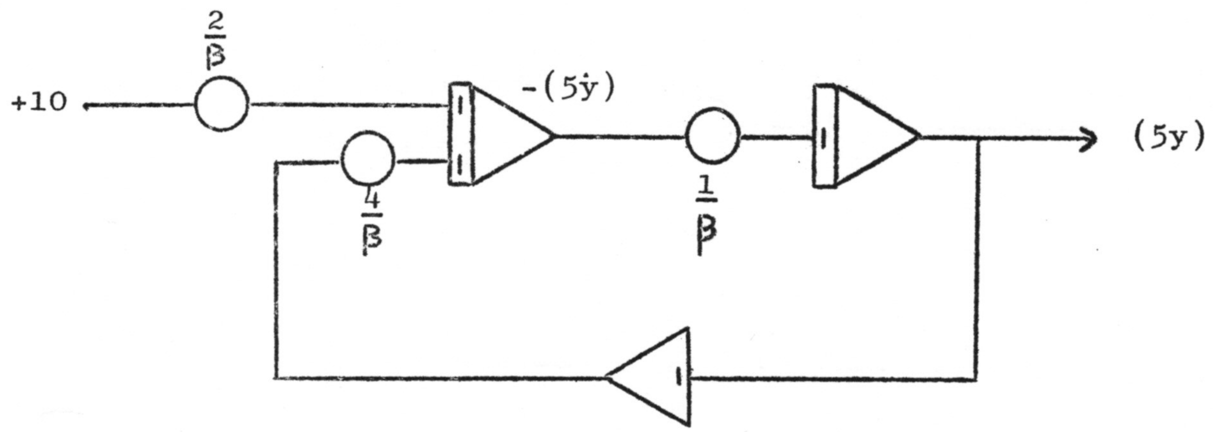
$$\frac{d(y)}{dt} = (\dot{y})$$

6.7.3. <u>Scaled Equations</u> (where $\Upsilon = \beta t$)

$$\frac{d(5\dot{y})}{d\Upsilon} = \frac{4}{\beta} \cdot 5.\frac{1}{10}.(10) - \frac{4}{\beta} \cdot (5y)$$

$$\frac{d(5y)}{d\Upsilon} = \frac{1}{\beta}.(5\dot{y})$$

6.7.4. <u>Flow Diagram</u> for generation of function y(t):



<u>Fig. 6.2.</u>

6.7.5. <u>Scaling the variables x and $\dot{x}$:</u>

Since the maximum value of y is 2, we may estimate the maximum values of x and $\dot{x}$ by means of examining the equation:

$$\ddot{x} + 2kx = 0, \text{ where } x = 1, \dot{x} = 0 \text{ at } t = 0.$$

This has a solution of the form $x = \cos\sqrt{2k}\,.\,t$ and so the maximum value of x is 1 and that of $\dot{x}$ is $\sqrt{10}$.

However, we appreciate that some of the solutions of the problem are unstable and because of this know that we should use, in our scaling, a higher maximum value for x than that predicted by calculation:

Let this maxinum value be 5. This means that the maximum for $\dot{x}$ is now $5.\sqrt{10}$, say 20.

Scaling

Scaled variables are $\left(\frac{x}{5} \cdot 10\right) = (2x)$

and $\left(\frac{\dot{x}}{20} \cdot 10\right) = \left(\frac{\dot{x}}{2}\right)$

The Standardised Equations for the complete system are:

$$\frac{d(\dot{x})}{dt} = -k(y).(x)$$

and $$\frac{d(x)}{dt} = (\dot{x})$$

6.7.6. Scaled Equations (where $\tau = \beta.t$)

$$\frac{d}{d\tau}\left(\frac{\dot{x}}{2}\right) = -\frac{k}{\beta} \cdot \frac{1}{2} \cdot \frac{1}{10} \cdot (5y).(2x)$$

$$\frac{d(2x)}{d\tau} = \frac{4}{\beta} \cdot \left(\frac{\dot{x}}{2}\right)$$

6.7.7. Flow Diagram

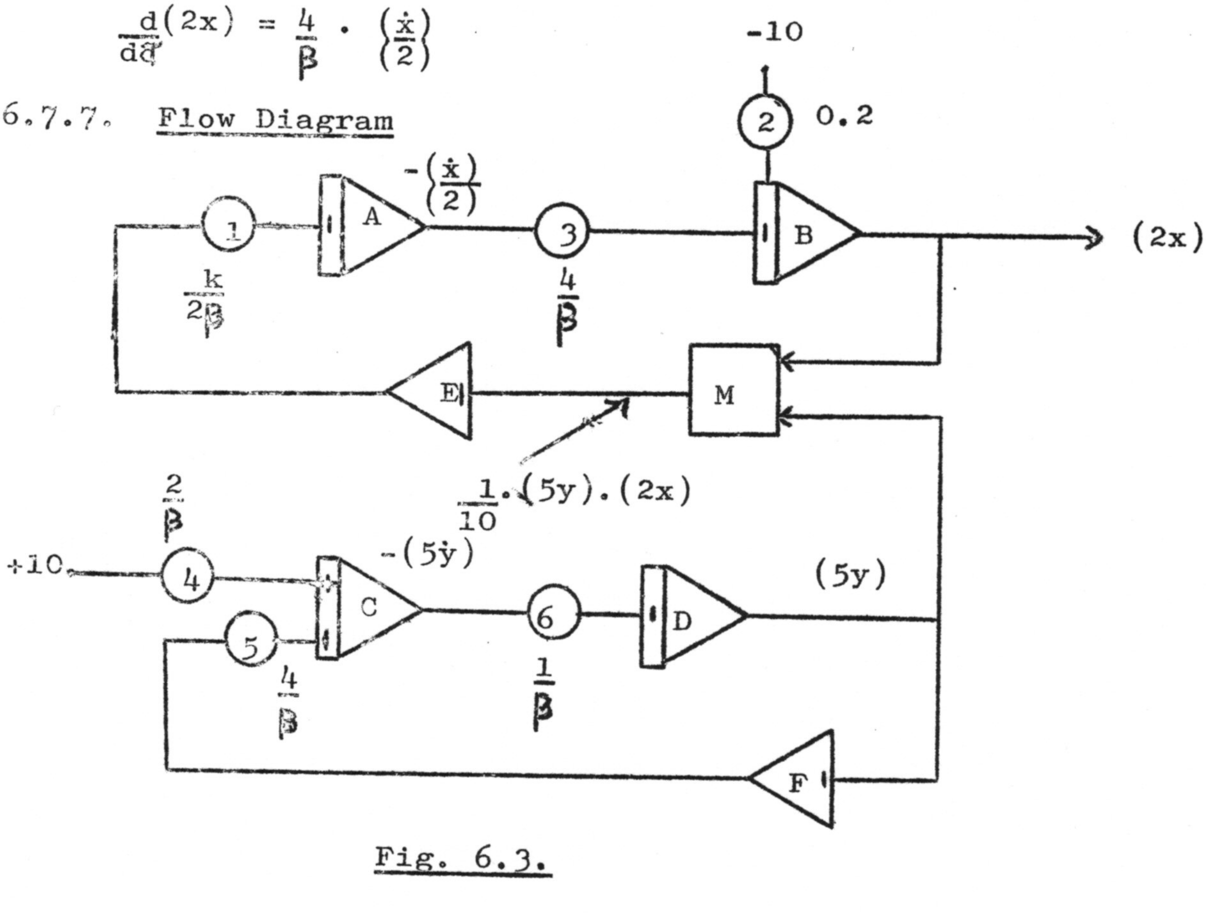


Fig. 6.3.

Once again, assignment and check sheets are left as an exercise for the reader.

6.7.8. Potentiometer Settings

Ref. No.	Parameter and Value	Coefficient ($\beta = 1$)
1	$\frac{k}{2\beta}$	$\frac{k}{2}$
2		0.2
3	$4/\beta$	4
4	$2/\beta$	2
5	$4/\beta$	4
6	$1/\beta$	1

Notes:
a) Since $\beta = 1$ the problem is being solved in real time.
b) Potentiometer 6 is not required.
c) Potentiometers 3,4 and 5 are all fed into 10x inputs, (one external resistor of 0.1MΩ is of course required for amplifier C to do this) their practical coefficients then being 0.4, 0.2 and 0.4 respectively. Potentiometer 1 has to be treated in a similar way when k has a value greater than 2.

It is suggested that k should be increased from a low value, say 0.2, in order to investigate the onset of instability in the system.

6.7.9. The Patching Diagram

Switches: Multiplier Unit to Compute and X. Amplifiers A,B,C and D all to $\int$.

Ae - Ad
Ah - Am
Ak - An
Au - Av
As - Fr
At - Eg
Ac - Er
Ar - Cc
Bh - Bm
Bk - Bn

Bd - Bj
Bb - Bc
Bg - Cd
Br - Mx
Ch - Cm
Ck - Cn
Cg - Dd
Ch - Ej
Cr - De
Dh - Dm

Dk - Dn
Da - Dc
Dr - Fe
Dr - My
Eh - Em
Em - Eq
Ee - Mz
Fh - Fm
Fm - Fq
Gm - Gq
Br - Display (variable x)